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Study of Magneto-Convection in a Square Cavity with a Central Spherical heat Source

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ABSTRACT

The aim of the present study is to investigate numerically the effect of direction of external uniform magnetic field on natural convection flow in a square cavity in the presence of a spherical heat source. The left wall is heated with a uniform temperature, while the rest walls are kept adiabatic and a circular heat source with higher temperature is placed at the centre. The effect of orientation of magnetic field on streamlines, isotherms, rate of heat transfer, average temperature and average velocity is studied. The governing equations along with boundary conditions are solved by using the finite element method based on Galarkin's weighted residual approach. The result indicates that the horizontal direction of imposed external magnetic field is most appropriate to reduce the fluid flow and heat transfer significantly.

Keywords: Magnetic field; Natural Convection; Cavity; Finite element method.

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1 INTRODUCTION

Natural convection in cavities has been an important topic in many engineering applications such as electronic equipment cooling, solar energy collectors, heat transfer in buildings and nuclear reactor etc. Heater plays an important role on the fluid flow and heat transfer in many engineering applications, particularly, electronic equipment cooling. Ho and Chang [1] investigated both numerically and experimentally natural convection heat transfer in a rectangular enclosure with four discrete heaters. Yucel and Turkoglu [2] studied natural convection in a rectangular enclosure with partial heating and cooling. They found that the mean Nusselt number decreases on increasing heater size for a given cooler size as well as the Nusselt number increases on increasing the cooler size for given heater size. A numerical study on combined convection in a partially heated lid-driven enclosure was made by Oztop [3]. He found that the location of the heater is the most effective parameter on combined convection flow and temperature field. Sankar and Venkatachalappa [4] studied the effect of surface tension on buoyancy driven convection in a vertical cylindrical annular cavity filled with a low Prandtl number fluid. They found that the heat transfer rate increases with Marangoni number. Chen and Chen [5] investigated numerically natural convection flows in a square enclosure with partially heated on the left and bottom walls. They revealed that the heat transfer rate is gradually increased on increasing the length of the heat source segment.Ghasemi and Aminossadati [6] numerically studied mixed convection in a partially heated square cavity. The results indicated that the direction and magnitude of the sliding wall velocity affect the heat transfer rate. Corcione and Habib [7] investigated natural convection in an inclined square cavity cooled at one side and partially heated at the opposite side. They observed that the average Nusselt number increases on increasing the Rayleigh and Prandtl numbers, as well as on increasing the dimensionless size of the heater. Hussain et al. [8] numerically studied natural convection in a square inclined enclosure with corrugated sidewalls. They found that the thermal boundary layers increase and concentrate near the heat source on increasing the inclination angle and Grashof number.

Magneto-hydrodynamics deals with electrically conducting fluid which plays a significant role in the field of metallic materials, agriculture, engineering and petroleum industries. Many researchers considered the natural convection problems in the presence of magnetic field. An analytical solution to the equations of magneto-hydrodynamics that can be used to model the effect of a transverse magnetic field on the buoyancy driven convection in a cavity was proposed by Garandet *et al.* [9]. Rudraiah *et al.* [10] numerically investigated the effect of a magnetic field on natural convection in a rectangular cavity filled with an electrically conducting fluid. Their findings were that the effect of the magnetic field decreased the rate of convective heat transfer. Khanafer and Chamkha [11] numerically studied hydromagnetic natural convection of a heat generating fluid in an inclined square porous cavity saturated with an electrically conducting fluid. They found that the effects of the magnetic field and the porous medium reduce the heat transfer and fluid circulation. Natural convection of a liquid metal in a laterally and volumetrically heated square cavity under the influence of the magnetic field is presented numerically by Sarris *et al.* [12]. They found that the flow oscillations are reduced or vanished on increasing the Hartmann numbers due to the magnetic field damping effect. Hossain *et al.* [13] studied the combined buoyancy and thermo-capillary convection flow of an electrically conducting fluid in an enclosure under an externally imposed magnetic field with internal heat generation. They found that changing the direction of the external magnetic field from horizontal to vertical leads to decrease in flow rates in both the primary and the secondary cells and that causes an increase in the effect of the thermo-capillary force.

Sankar *et al.* [14] examined the effect of magnetic field on the buoyancy-driven convection in a vertical cylindrical annulus. They found that the heat transfer rate increases with radii ratio and decreases with the Hartmann numbers. Sivasankaran and Ho [15] studied the effect of variable fluid properties on magneto-convection of water near its density maximum in an enclosure. They found that the average Nusselt number decreases on increasing the Hartmann number. Pirmohammadi and Ghassemi [16] investigated natural convection flow in the presence of a magnetic field in a tilted enclosure heated from below and cooled from top. They found that the heat transfer mechanisms and the flow characteristics inside the enclosure depend strongly on both the strength of the magnetic field and inclination angle. Mamun et al. [17] numerically investigated the effects of magnetic force on natural convection in a porous trapezoidal enclosure saturated with an electrically conducting fluid. They found that the optimum heat transfer rate at higher values of modified Raleigh number in the absence of magnetic force. Mansour et al. [18] numerically investigated the magneto-convection in an inclined square porous cavity with internal heat generation. Parvin and Nasrin [19] made an analysis on convection heat transfer and fluid flow in a square cavity with a horizontal circular heated obstacle. It is observed that the diameter of the body has a significant effect on the flow and temperature fields.

The present study is conducted to understand the effect of direction of magnetic field on natural convection flow and heat transfer in a square cavity having a circular heater at the centre.

2 MATHEMATICAL FORMULATIONS

Steady, laminar, incompressible free convective flow and heat transfer in a two dimensional square cavity of length L is considered. The top, bottom and right walls of the cavity are well insulated while the left wall is maintained at a temperature T_c. A circular heated body of diameter d heated with the temperature T_h is located at the centre of the cavity. The cavity is filled with an electrically conducting fluid. The physical model and coordinate system considered in the investigation is shown in **Fig.1**. The gravity acts in the negative y-direction. The thermo-physical properties of the fluid are assumed to be constant. The Boussinesq approximation is valid. It is also assumed that the uniform magnetic field $B= B_x e_x + B_y e_y$ of constant magnitude $B_0 = \sqrt{B_x^2 + B_y^2}$ is applied, where e_x and e_y are unit vectors in the Cartesian coordinate system. The orientation of the magnetic field form an angle ψ with horizontal axis such that $\tan\psi = B_x / B_y$. The viscous dissipation and Joule heating are neglected in the study. The governing equations of mass, momentum and energy can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \ldots \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{\sigma B_0^2}{\rho}\left(v\sin\psi\cos\psi - u\sin^2\psi\right) \quad \ldots \quad \ldots \quad \ldots$$

(2)

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\beta(T - T_c) + \frac{\sigma B_0^2}{\rho}\left(u\sin\psi\cos\psi - v\cos^2\psi\right)$$

$$\ldots \quad (3)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \quad \ldots \quad (4)$$

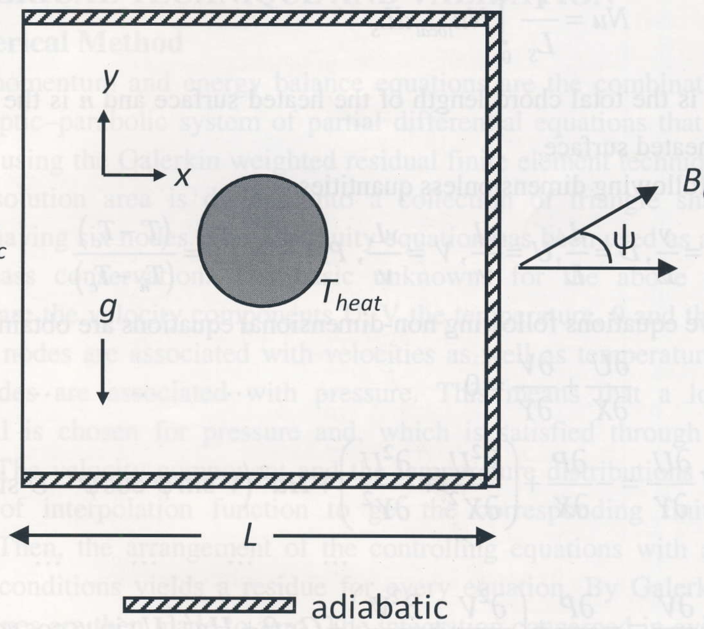

Figure 1: Schematic diagram of the problem.

The boundary conditions for the present problem can be written as follows:

at the left wall: $u = v = 0, T = T_c$

at the circular body surface : $u = 0,\ v = 0, T = T_h$

at the rest walls: $u = 0, v = 0, \frac{\partial T}{\partial n} = 0$

The rate of heat transfer is computed at the bottom wall and is expressed in terms of the local Nusselt number as $Nu_{local} = -\frac{\partial T}{\partial n} L$ where, h represents the heat transfer coefficient, k thermal conductivity and n the coordinate direction normal to the surface. The dimensionless normal temperature gradient can be written as

$$\frac{\partial T}{\partial n} = \sqrt{\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2}$$

while the average Nusselt number (Nu) is obtained by integrating the local Nusselt number along the heated surface and is defined by

$$Nu = \frac{1}{L_S}\int_0^{L_s} Nu_{local} dL_S$$

where L_S is the total chord length of the heated surface and n is the coordinate along the heated surface.

Using the following dimensionless quantities

$$X = \frac{x}{L}, Y = \frac{y}{L}, D = \frac{d}{L}, U = \frac{ul}{\nu}, V = \frac{\nu L}{\nu}, P = \frac{pL^2}{\rho \nu^2}, \theta = \frac{(T - T_c)}{(T_h - T_c)}$$

in the above equations following non-dimensional equations are obtained

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) + Ha^2\left(V \sin\psi \cos\psi - U \sin^2\psi\right) \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + Gr\,\theta + Ha^2\left(U \sin\psi \cos\psi - V \cos^2\psi\right) \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{Pr}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \quad (8)$$

Here $Gr = \frac{g\beta\Delta T L^3}{\nu^2}$, $Pr = \frac{\nu}{\alpha}$ and $Ha^2 = \frac{\sigma B_0^2 L^2}{\mu}$ are Grashof number, Prandtl number and square of Hartmann number respectively.

The boundary conditions for the present problem are specified as follows:

At the left wall: $U = V = 0, \theta = 0$

At the circular body surface: $U = V = 0, \theta = 1$

At the rest walls: $U = V = 0, \frac{\partial \theta}{\partial N} = 0$

The average Nusselt number at the heated surface may be expressed as

$$Nu_{av} = -\frac{1}{S}\int_0^S \frac{\partial \theta}{\partial N} dS$$

where S is the non dimensional length along the heated surface.

3 NUMERICAL TECHNIQUE AND VALIDATION

3.1 Numerical Method

The momentum and energy balance equations are the combinations of the mixed elliptic–parabolic system of partial differential equations that have been solved by using the Galerkin weighted residual finite element technique [20]. At first the solution area is divided into a collection of triangle shaped finite elements having six nodes. The continuity equation has been used as a constraint due to mass conservation. The basic unknowns for the above differential equations are the velocity components U, V the temperature, θ and the pressure, P. All six nodes are associated with velocities as well as temperature; only the corner nodes are associated with pressure. This means that a lower order polynomial is chosen for pressure and, which is satisfied through continuity equation. The velocity component and the temperature distributions are written in terms of interpolation function to get the corresponding finite element equation. Then, the arrangement of the controlling equations with appropriate boundary conditions yields a residue for every equation. By Galerkin method these residues are then made to zero. The integration concerned in every term of these equations is carried out by Gauss's quadrature scheme. The set of non-linear algebraic Equations are then solved using reduced integration technique and Newton–Raphson method [21].

At first the solution area is divided to a collection of triangle shaped finite elements having six nodes. After that dependent variables within each element are approximated by applying interpolation functions. Then, the arrangement of the controlling equations with appropriate conditions to boundaries yields a residue for every equation. By Galerkin method these residues are then made to zero. The integration concerned in every term of these equations is carried out by Gauss's quadrature scheme. The highly coupled non-linear equations so acquired are customized by implementation of boundary conditions and converted to linear algebraic equations applying Newton's technique. At last, solutions of these linear equations are obtained through Triangular Factorization method.

3.2 Grid Generation

Grid or mesh generation is the partition of the geometry model into small units of simple shapes named finite elements, control volume etc that approximates the physical domain in finite element method. Dependent variables are approximated at the local element coordinates defined by the numerical grid. It is mainly a disconnected demonstration of the physical domain where the

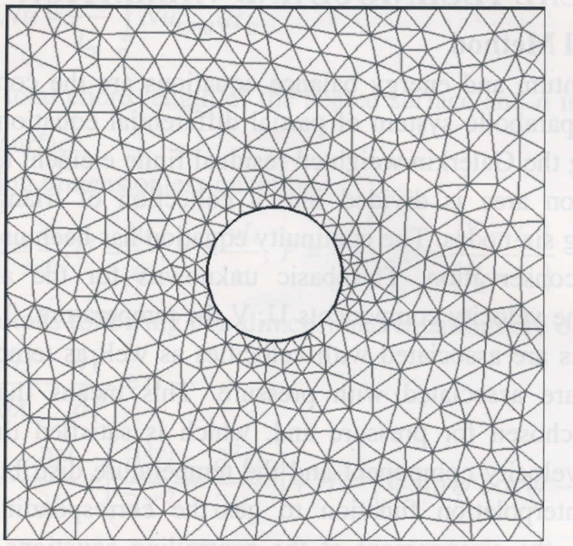

Figure 2: Mesh generation of the physical domain.

solutions are to be carried out. Meshing the complicated geometry make the finite element method a powerful technique to solve boundary value problems occurring in a range of engineering applications. **Fig.2** shows the mesh configuration of present physical domain with triangular finite elements.

3.3 Validation

The validation of present computational results is made against the existing results for natural convection in a square cavity in the presence of the horizontal uniform magnetic field [15] and is shown in Table 1. A good agreement is obtained between the present results and previous results. Hence, these results provide confidence in the accuracy of the present code to study the problem considered here.

4 RESULTS AND DISCUSSION

Numerical simulation on convective flow and heat transfer in a square cavity with a heat source is investigated. The working fluid is chosen as electrically conducting fluid with Prandtl number Pr = 0.71. The effects of four different directions of the external magnetic fields: horizontal ($\psi = 0^0$), inclined ($\psi = 30^0$, $\psi = 60^0$) and vertical direction ($\psi = 90^0$) are chosen to study the effect of direction of the external magnetic field on fluid flow.

4.1 Effect on Streamlines and isotherms

Fig.3 depicts the effect of different orientation of magnetic field on the fluid flow. At $\psi = 0^0$, the flow consists of a counterclockwise rotating cell having two small inner vortices near to the heating location and occupies the whole cavity. These two eddies are changed to a single and then extended with elevating ψ. When increasing the angles sequentially from 0^0 to 90^0 there is a considerable change in flow pattern. It is interesting to note that the cell elongated diagonally for the inclination of magnetic field. At $\psi = 30^0$, another eddy is formed elliptically towards the top corner which changes its shape, size and direction for increasing ψ. When changing the angle of external magnetic field to vertical, the flow pattern is changed differently and the cells are further extended.. The influence of the magnetic field on flow pattern is apparent from these figures. This is due to the retarding effect of Lorentz force.

The influence of the direction of the external magnetic field on temperature field is shown in **Fig.4**. The temperature field is not affected much by the orientation of the magnetic field like the flow field. The thermal boundary layer become slightly thick and the isotherms are more concentrated for $\psi = 60^0$ and $\psi = 90^0$ than the other values of ψ.

4.2 Effect on heat transfer rate

The effect of orientation of the external magnetic field along with Grashof Number *Gr* on average Nusselt number *Nu* is displayed in **Fig.5**. The *Nu* slightly changes on changing the orientation of the external magnetic field because the temperature field is not affected much with the orientation of magnetic field like flow field. It is also observed that the heat transfer rate is increased on increasing the Grashof Number. It can be seen from the figure that there is a small change in average heat transfer rate when changing the direction of external magnetic field. Among the four directions, inclined magnetic fields ($\psi = 30^0$ and $\psi = 60^0$) produce higher heat transfer rate.

4.3 Effect on mean temperature and mean velocity

Fig.6 shows the variation in average fluid temperature θ_{av} in the cavity due to changing direction of external magnetic field with *Gr*. θ_{av} decreases with increasing *Gr* and ψ. Lower average temperature is observed for $\psi = 30^0$ and $\psi = 60^0$.

Average velocity in the computational domain for different direction of external magnetic field wit h *Gr* is displayed in **Fig.7**. The highest average

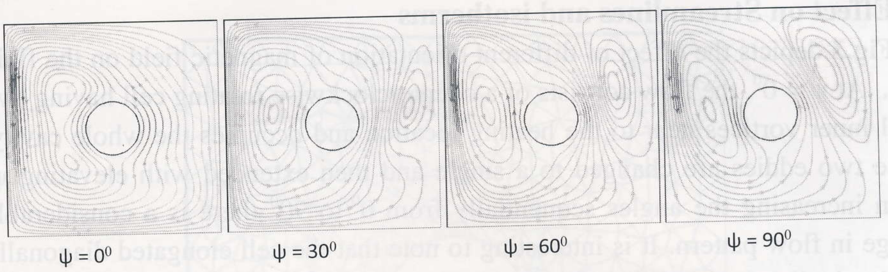

Figure 3: Streamlines for different angles of magnetic field made with horizontal direction.

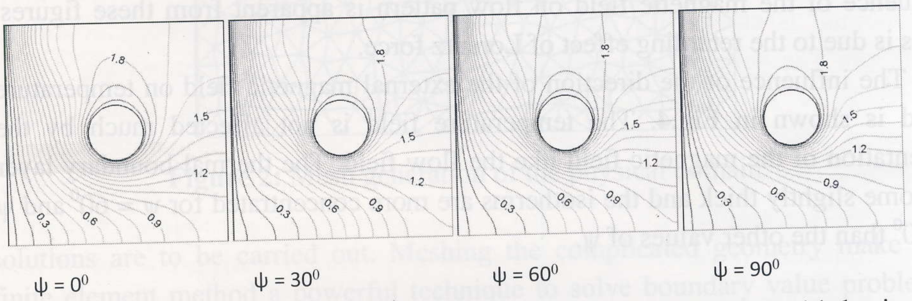

Figure 4: Isotherms for different angles of magnetic field made with horizontal direction.

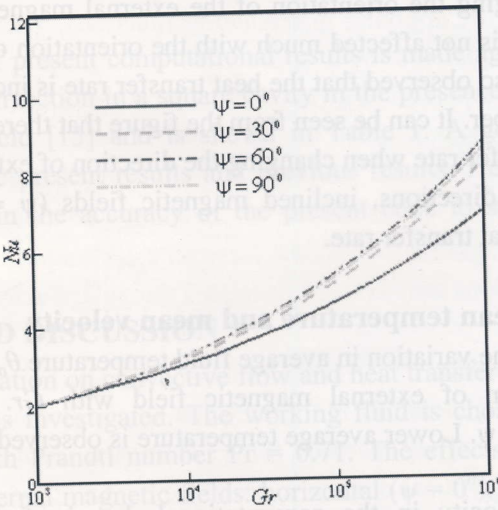

Figure 5: Average Nusselt number for different ψ along with *Gr*.

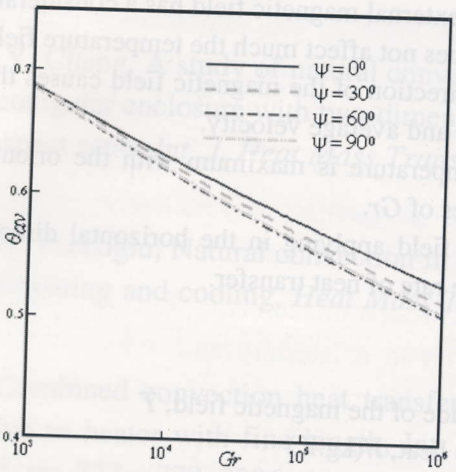

Figure 6: Average bulk temperature for different ψ along with *Gr* .

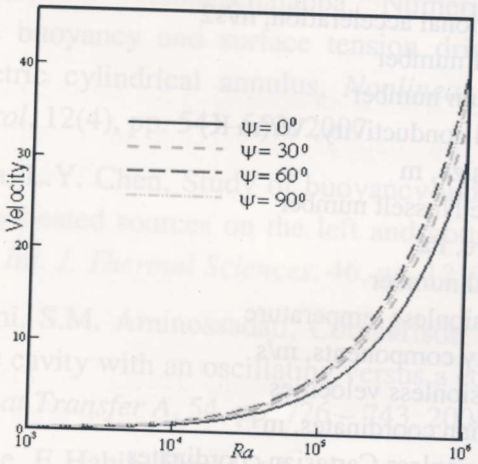

Figure 7: Average velocity of the fluid for different ψ along with *Gr*.

velocity is observed for $\psi = 60^0$. The velocity is lowest when the magnetic field is applied to the horizontal direction.

5 CONCLUSIONS

A numerical simulation is performed to analyze the magneto-convection in a square cavity having a circular heat source with different orientation of imposed magnetic field. The following results are concluded in the study:

(i) The direction of external magnetic field has a considerable effect on flow structure but it does not affect much the temperature field.

(ii) The horizontal direction of the magnetic field causes the lowest average heat transfer rate and average velocity.

(iii) The average temperature is maximum with the orientation $\psi = 0^0$ for considered values of *Gr*.

Therefore the magnetic field applying in the horizontal direction will be most appropriate to reduce the rate of heat transfer.

NOMENCLATURE

B_0	magnitude of the magnetic field, *T*
c_p	specific heat, J/(kg.K)
d	diameter of the heat source, m
D	dimensionless diameter of the heat source
g	gravitational acceleration, m/s2
Gr	Grashof number
Ha	Hartmann number
k	thermal conductivity, W/(m K)
L	cavity size, m
Nu	average Nusselt number
P	pressure, Pa
Pr	Prandtl number
T	dimensionless temperature
u, v	velocity components, m/s
U, V	dimensionless velocities
x, y	Cartesian coordinates, m
X, Y	dimensionless Cartesian coordinates
Greek symbols	
α	thermal diffusivity, m^2/s
β	coefficient of thermal expansion, K^{-1}
θ	temperature, K
ψ	orientation of the magnetic field
ρ	density, kg/m^3
μ	dynamic viscosity, Pa s
ν	kinematic viscosity, m^2/s
Subscripts	
c	cold
h	heater

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