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MHD Free Convection Flow along a Vertical Wavy Surface with Linear Variation of Thermal Conductivity and Reciprocal Variation of Viscosity with Temperature

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ABSTRACT

The present work describes the effect on MHD free convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface with temperature dependent thermal conductivity and viscosity. Viscosity considered is inversely proportional to the linear function of temperature. The governing boundary layer equations are first transformed into a non-dimensional form using suitable set of dimensionless variables. The resulting nonlinear system of partial differential equations are mapped into the domain of a vertical flat plate and then solved numerically employing the implicit finite difference method, known as the Keller-box scheme. The numerical results of the skin friction coefficient and the rate of heat transfer in terms of local Nusselt number, the stream lines as well as the isotherms are shown graphically for a selection of parameters set consisting of thermal conductivity variation parameter and viscosity variation parameter .

Keywords: MHD, Convection flow, Wavy surface, Keller-box scheme, Stream line, Thermal conductivity.

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1 INTRODUCTION

Many investigators because of its considerable practical applications have presented the laminar free convection flow of an electrically conducting fluid and heat transfer problem. Physical properties like viscosity and thermal conductivity mast be changed significantly with temperature. The viscosity of liquids decreases and the viscosity of gases increase with temperature. It is also necessary to study the heat transfer from an irregular surface because irregular surfaces are often present in many applications, such as radiator, heat exchangers and heat transfer enhancement devices. The viscosity and thermal conductivity of the fluid to be proportional to a linear function of temperature, two semi-empirical formulae which was proposed by Charraudeau [1]. Yao [2] first investigated the natural convection heat transfer from an isothermal vertical wavy surface and used an extended Prantdl's transposition theorem and a finite-difference scheme. He proposed a simple transformation to study the natural convection heat transfer for an isothermal vertical sinusoidal surface. These simple coordinate transformations method to change the wavy surface into a flat plate. Alam et al. [3] considered the problem of free convection from a wavy vertical surface in presence of a transverse magnetic field. On the other hand, the combined effects of thermal and mass diffusion on the natural convection flow of a viscous incompressible fluid along a vertical wavy surface have been investigated by Hossain and Rees [4]. In this paper the effect of waviness of the surface on the heat and mass flux has been investigated in combination with the species concentration for a fluid having Prandtl number equal to 0.7. Hossain and Munir [5] investigated the natural convection flow of a viscous fluid about a truncated cone with temperature dependent viscosity and thermal conductivity. Natural convection of fluid with temperature dependent viscosity from heated vertical wavy surface has been investigated by Hossain et al. [6]. Munir et al. [7] considered natural convection of a viscous fluid with viscosity inversely proportional to linear function of temperature from a vertical wavy cone. Natural convection heat and mass transfer along a vertical wavy surface have been investigated by Jang et al. [8]. Molla et al. [9] studied natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation/absorption. Yao [10] also studied natural convection along a vertical complex wavy surface. Very recently, Parveen and Alim [11] investigated Joule heating effect on Magnetohydrodynamic natural convection flow along a vertical wavy surface with viscosity dependent on temperature. At the same time Parveen and Alim [12] considered effect of temperature dependent thermal conductivity on magnetohydrodynamic natural convection flow along a vertical wavy surface. It is known that thermal conductivity may be change significantly with temperature. For a liquid, it has been found that the thermal conductivity k varies with temperature in an approximately linear manner in the range from 0 to 400^0 F (see Kays [13]).

In all the above investigations the effect on MHD free convection flow along a uniformly heated vertical wavy surface with temperature dependent thermal conductivity and viscosity as the inversely proportional to linear function of temperature has not been considered. The present study is used to deal with this problem. Thermal conductivity of the fluid considered proportional to a linear function of temperature. The governing partial differential equations are reduced to locally non-similar partial differential forms by adopting some appropriate transformations. The transformed boundary layer equations are solved numerically using implicit finite difference scheme together with the Keller box technique [14]. Numerical results of the surface shear stress in terms of local skin friction coefficient and the rate of heat transfer in terms of local Nusselt number, the streamlines as well as the isotherms are presented graphically.

2 FORMULATION OF THE PROBLEM

Consider a steady two dimensional free convection flow of an electrically conducting viscous and incompressible fluid with variable thermal conductivity and viscosity along a vertical wavy surface. Over the work it is assumed that the surface temperature of the vertical wavy surface T_w is uniform, where $T_w > T_\infty$. The boundary layer analysis outlined below allows ⊠(X) being arbitrary, but our detailed numerical work assumed that the surface exhibits sinusoidal deformations. The wavy surface may be described by

$$Y_w = \bar{\sigma}(X) = \alpha \sin\left(\frac{n\pi X}{L}\right) \tag{1}$$

where L is the wavelength associated with the wavy surface.

The geometry of the wavy surface and the two-dimensional cartesian coordinate system are shown in Fig. 1.

The conservation equations for the flow characterized with steady, laminar and two-dimensional boundary layer, under the usual Boussinesq approximation, the continuity, momentum and energy equations can be written as:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{2}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{1}{\rho}\frac{\partial P}{\partial X}+\frac{1}{\rho}\nabla.(\mu\nabla U)+g\beta(T-T_\infty)-\frac{\sigma_0\beta_0^2}{\rho}U \tag{3}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{1}{\rho}\frac{\partial P}{\partial Y}+\frac{1}{\rho}\nabla.(\mu\nabla V) \tag{4}$$

$$U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{\rho C_p}\nabla.(k\nabla T) \tag{5}$$

where (X, Y) are the dimensional coordinates along and normal to the tangent of the surface and (U, V) are the velocity components parallel to (X,Y), $\nabla^2(=\partial^2/\partial x^2+\partial^2/\partial y^2)$ is the Laplacian operator, g is the acceleration due to

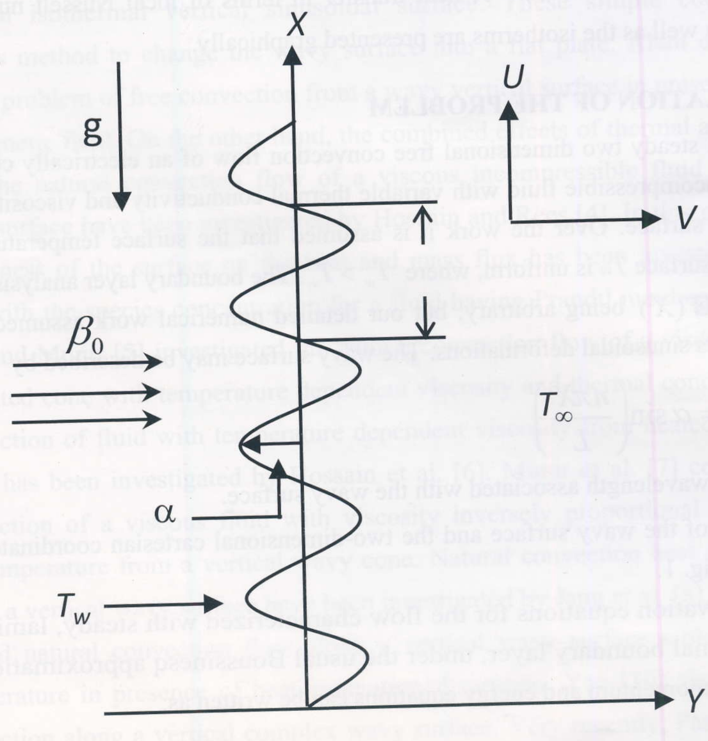

Figure 1: The coordinate system and the physical model.

gravity, P is the dimensional pressure of the fluid, ρ is the density, β_0 is the strength of magnetic field, σ_0 is the electrical conduction, $k(T)$ is the thermal conductivity of the fluid in the boundary layer region depending on the fluid temperature, β is the coefficient of thermal expansion, ν ($= \mu/\rho$) is the kinematics viscosity, $\mu(T)$ is the dynamic viscosity of the fluid in the boundary layer region depending on the fluid temperature and C_p is the specific heat due to constant pressure.

The boundary conditions relevant to the above problem are

$$U = 0, V = 0, T = T_w \quad at\, Y = Y_w = \bar{\sigma}(X) \tag{6a}$$

$$U = 0, \quad T = T_\infty, \quad P = P_\infty \quad as \quad Y \to \infty \tag{6b}$$

where T_w is the surface temperature, T_∞ is the ambient temperature of the fluid and P_∞ is the pressure of fluid outside the boundary layer.

The variable thermal conductivity chosen in this study that is introduced by Charraudeau [1] and used by Hossain and Munir [5] as follows:

$$k = k_\infty\left[1 + \gamma^*(T - T_\infty)\right] \tag{7}$$

Temperature dependent viscosity inversely proportional to linear function of temperature chosen is this case, which is introduced by Hossain and Munir [5] as follows:

$$\mu = \frac{\mu_\infty}{1 + \varepsilon^*(T - T_\infty)} \tag{8}$$

where μ_∞ is the viscosity and k_∞ is the thermal conductivity of the ambient fluid, $\varepsilon^* = \frac{1}{\mu_f}\left(\frac{\partial \mu}{\partial T}\right)_f$ and $\gamma^* = \frac{1}{k_f}\left(\frac{\partial k}{\partial T}\right)_f$ is a constant evaluated at the film temperature of the flow $T_f = 1/2\,(T_w + T_\infty)$.

Using Prandtl's transposition theorem to transform the irregular wavy surface into a flat surface as extended by Yao [2] and boundary-layer approximation, the following dimensionless variables are introduced for non-dimensionalizing the governing equations,

$$x = \frac{X}{L}, \quad y = \frac{Y - \bar{\sigma}}{L} Gr^{\frac{1}{4}}, \; p = \frac{L^2}{\rho \nu^2} Gr^{-1} P$$

$$u = \frac{\rho L}{\mu} Gr^{-1/2} U, \quad v = \frac{\rho L}{\mu} Gr^{-1/4} (V - \sigma_x U), \tag{9}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \sigma_x = \frac{d\bar{\sigma}}{dX} = \frac{d\sigma}{dx}, \quad Gr = \frac{g\beta(T_w - T_\infty)}{\nu^2} L^3$$

where θ is the non-dimensional temperature function and $(u,\ v)$ are the dimensionless velocity components parallel to $(x,\ y)$. Introducing the above dimensionless dependent and independent variables into equations (2)–(5), the following dimensionless form of the governing equations are obtained after ignoring terms of smaller orders of magnitude in Gr, the Grashof number defined in (9).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{10}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + Gr^{1/4}\sigma_x \frac{\partial p}{\partial y} + \frac{(1+\sigma_x^2)}{(1+\varepsilon\theta)}\frac{\partial^2 u}{\partial y^2} - \frac{\varepsilon(1+\sigma_x^2)}{(1+\varepsilon\theta)^2}\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y} - Mu + \theta \tag{11}$$

$$\sigma_x\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -Gr^{1/4}\frac{\partial p}{\partial y} + \frac{\sigma_x(1+\sigma_x^2)}{(1+\varepsilon\theta)}\frac{\partial^2 u}{\partial y^2} - \frac{\varepsilon\sigma_x(1+\sigma_x^2)}{(1+\varepsilon\theta)^2}\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y} - \sigma_{xx}u^2 \tag{12}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\left(1+\sigma_x^2\right)\left(1+\gamma\theta\right)\frac{\partial^2 \theta}{\partial y^2} + \frac{1}{\Pr}\left(1+\sigma_x^2\right)\gamma\left(\frac{\partial \theta}{\partial y}\right)^2 \tag{13}$$

where $\Pr = \dfrac{C_p \mu_\infty}{k_\infty}$ is the Prandtl number, $M = \dfrac{\sigma_0 \beta_0^2 L^2}{\mu Gr^{1/2}}$ is the magnetic parameter, $\varepsilon = \varepsilon^*(T_w - T_\infty)$ is viscosity variation parameter and $\gamma = \gamma^*(T_w - T_\infty)$ is the thermal conductivity variation parameter.

It can easily be seen that the convection induced by the wavy surface is described by equations (10)–(13). We further notice that, equation (12) indicates that the pressure gradient along the y-direction is $O(Gr^{-1/4})$, which implies that lowest order pressure gradient along x -direction can be determined from the inviscid flow solution. For the present problem this pressure gradient ($\partial p/\partial x = 0$) is zero. Equation (12) further shows that $Gr^{1/4}\partial p/\partial y$ is $O(1)$ and is determined by the left-hand side of this equation. Thus, the elimination of $\partial p/\partial y$ from equations (11) and (12) leads to

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{(1+\sigma_x^2)}{(1+\varepsilon\theta)}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_x\sigma_{xx}}{1+\sigma_x^2}u^2-\frac{\varepsilon(1+\sigma_x^2)}{(1+\varepsilon\theta)^2}\frac{\partial u}{\partial y}\frac{\partial\theta}{\partial y} \\ -\frac{M}{1+\sigma_x^2}u+\frac{1}{1+\sigma_x^2}\theta \tag{14}$$

The corresponding boundary conditions for the present problem then turn into

$$\left.\begin{aligned} &u=v=0,\quad \theta=1 \quad at \quad y=0 \\ &u=\theta=0,\quad p=0 \quad as \quad y\to\infty \end{aligned}\right\} \tag{15}$$

Now we introduce the following transformations to reduce the governing equations to a convenient form:

$$\psi=x^{3/4}f(x,\eta),\quad \eta=yx^{-1/4},\quad \theta=\theta(x,\eta) \tag{16}$$

where η is the pseudo similarity variable and ψ is the stream function that satisfies the equation (10) and is defined by

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}. \tag{17}$$

Introducing the transformations given in Eq. (16) and into Eqs. (14) and (13) the following system of non linear equations are obtained,

$$\frac{(1+\sigma_x^2)}{(1+\varepsilon\theta)}f'''+\frac{3}{4}ff''-\left(\frac{1}{2}+\frac{x\sigma_x\sigma_{xx}}{1+\sigma_x^2}\right)f'^2+\frac{1}{1+\sigma_x^2}\theta-\frac{Mx^{1/2}}{1+\sigma_x^2}f' \\ -\frac{\varepsilon(1+\sigma_x^2)}{(1+\varepsilon\theta)^2}\theta' f''=x\left(f'\frac{\partial f'}{\partial x}-f''\frac{\partial f}{\partial x}\right) \tag{18}$$

$$\frac{1}{\Pr}\left(1+\sigma_x^2\right)\left(1+\gamma\theta\right)\theta''+\frac{1}{\Pr}\left(1+\sigma_x^2\right)\gamma\theta'^2+\frac{3}{4}f\theta'=x\left(f'\frac{\partial\theta}{\partial x}-\theta'\frac{\partial f}{\partial x}\right) \quad (19)$$

The boundary conditions (15) now take the following form:

$$\left.\begin{array}{ll} f(x,o)=f'(x,o)=0, & \theta(x,o)=1 \\ f'(x,\infty)=0, & \theta(x,\infty)=0 \end{array}\right\} \quad (20)$$

In the above equations prime denote the differentiation with respect to η.

The local skin friction coefficient C_{fx} and the rate of heat transfer in terms of the local Nusselt number Nu_x takes the following form:

$$C_{fx}(Gr/x)^{1/4}/2=\frac{\sqrt{1+\sigma_x^2}}{(1+\varepsilon)}f''(x,o) \quad (21)$$

$$Nu_x(Gr/x)^{-1/4}=-\left(1+\gamma\right)\sqrt{1+\sigma_x^2}\,\theta'(x,o) \quad (22)$$

3 NUMERICAL METHOD

Solutions of the local non-similar partial differential equations (18) and (19) together with the boundary condition (20) are solved numerically by using implicit finite difference method with the Keller-box Scheme. Since a good description of this method and its application to the boundary layer flow problems are given in the book by Cebeci and Bradshaw [14] and broadly used by Hossain et al. [3-6].

The discretization of momentum and energy equations are carried out with respect to non-dimensional coordinates x and η to convey the equations in finite difference form by approximating the functions and their derivatives in terms of the central differences in both the coordinate directions. Then the required equations are to be linearized by using Newton's quasi-linearization method. The Jacobian matrix has a block-tridiagonal structure and the difference equations are efficiently solved using a block-matrix version of the Thomas algorithm.

4 RESULTS AND DISCUSSION

The present work is to analyze the effect on MHD free convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface with temperature dependent thermal conductivity and viscosity as the inversely

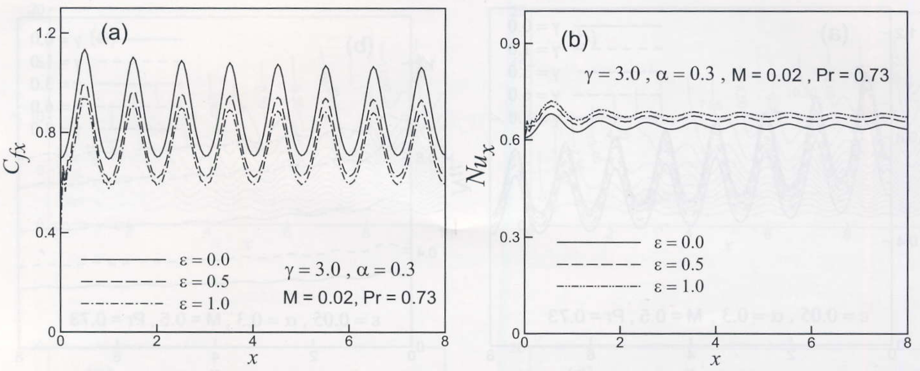

Figure 2: Variation of (a) skin friction coefficient C_{fx} and (b) rate of heat transfer Nu_x against x for different values of ε with $\alpha = 0.3$, $M = 0.02$, $\gamma = 3.0$ and Pr = 0.73.

proportional to linear function of temperature. Numerical values of local shearing stress and the rate of heat transfer are calculated from equations (21) and (22) in terms of the skin friction coefficients C_{fx} and Nusselt number Nu_x respectively for a wide range of the axial distance x. For different values of the aforementioned parameters γ and ε, the skin friction coefficient C_{fx}, the rate of heat transfer in terms of Nusselt number Nu_x, the streamlines as well as the isotherms are shown graphically in Figs. 2-7.

Figs. 2(a) and 2(b) show that increase in the value of $\varepsilon = (0.0, 0.5, 1.0)$ leads to decrease the value of skin friction coefficient and increases the rate of heat transfer in terms of the local Nusselt number Nu_x while Prandtl number Pr = 0.73, $\alpha = 0.3$, $M = 0.02$ and $\gamma = 3.0$ at different position of x. Moreover, the maximum values of local skin friction coefficient C_{fx} are 1.13392, 0.99176 and 0.93543 for $\varepsilon = 0.0, 0.5, 1.0$ respectively which occurs at different position of x. Furthermore, maximum values of local the rate of heat transfer are 0.68096, 0.70559 and 0.72124 for $\varepsilon = 0.0, 0.5, 1.0$ respectively which occurs at $x = 0.55$. It is seen that the local skin friction coefficient C_{fx} decreases by 17.50% and local the rate of heat transfer increases by 5.58% respectively. Here it is concluded that for high viscous fluid when inversely proportional to linear function of temperature then the skin friction coefficient is slow and the corresponding rate of heat transfer is higher.

The analysis of the effect of thermal conductivity parameter γ = (0.0, 1.0, 3.0, 6.0 and 8.0) on the surface shear stress in terms of the local skin friction coefficient C_{fx} and the rate of heat transfer in terms of the local Nusselt number

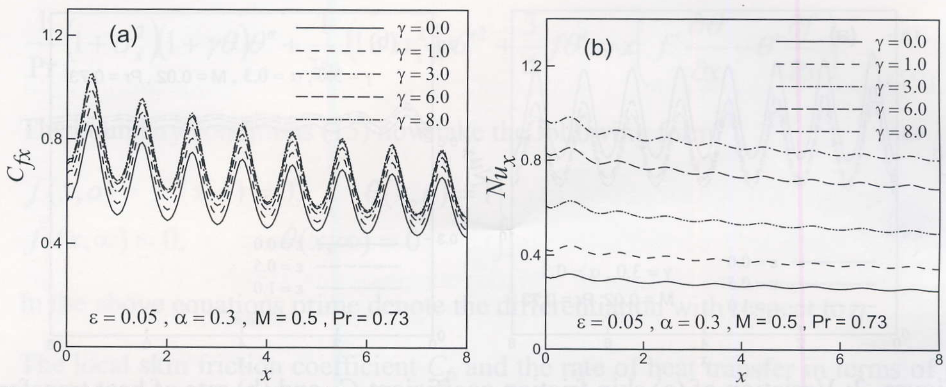

Figure 3: Variation of (a) skin friction coefficient C_{fx} and (b) rate of heat transfer Nu_x against x for different values of γ with $\alpha = 0.3$, $\varepsilon = 0.05$, $M = 0.5$ and $\Pr = 0.73$.

Nu_x against x are exposed within the boundary layer with $\alpha = 0.3$, $M = 0.5$, $\varepsilon = 0.05$ and $\Pr = 0.73$ in Fig. 3. From Fig. 3 it is noted that for increasing values of thermal conductivity parameter γ, both the skin friction coefficient and the heat transfer rates increase along the upstream direction of the surface. Increasing values of thermal conductivity parameter γ increase the velocity and the temperature as well as the temperature gradient of the surface.

Figs. 4 and 5 illustrate the effect of viscosity parameter ε on the development of streamlines and isotherms profile which are plotted for the amplitude of the wavy surface $\alpha = 0.3$, Prandtl number $\Pr = 0.73$, $\gamma = 3.0$ and $M = 0.02$. The maximum values of ψ, that is, $\psi_{\max}$ are 12.78, 13.17, 13.25 and 13.29 for viscosity parameter $\varepsilon = 0.0$, 0.50, 1.0 and 2.0 respectively. It is observed from Fig. 4 that as the values of ε increases the velocity boundary layer becomes higher gradually and the opposite results observed for the thermal boundary layer from Fig. 5.

The effect of thermal conductivity parameter γ equal to 0.0, 3.0, 6.0 and 8.0 the streamlines and isotherms profile are depicted by the Figs. 6 and 7 respectively while Prandtl number $\Pr = 0.73$, amplitude of wavy surface $\alpha = 0.3$, viscosity parameter $\varepsilon = 0.05$ and magnetic parameter $M = 0.5$. Fig. 6 depicts that the maximum values of ψ increases steadily while the values of γ increases. The maximum values of ψ, that is, $\psi_{\max}$ is 6.22 for $\gamma = 0.0$, $\psi_{\max}$ is 9.32 for $\gamma = 3.0$, $\psi_{\max}$ is 11.56 for $\gamma = 6.0$ and $\psi_{\max}$ is 12.26 for $\gamma = 8.0$. From Fig. 7, it is noted that the temperature of the fluid flow increases for increasing values of γ.

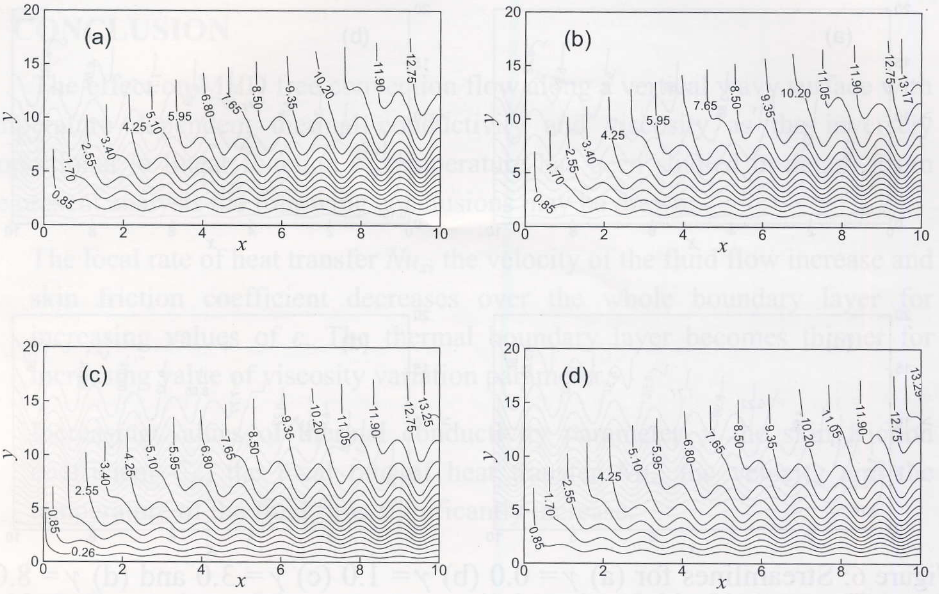

Figure 4: Streamlines for (a) $\varepsilon = 0.0$ (b) $\varepsilon = 0.5$ (c) $\varepsilon = 1.0$ and (d) $\varepsilon = 2.0$ while $\alpha = 0.3$, $M = 0.02$, $\gamma = 3.0$ and Pr = 0.73.

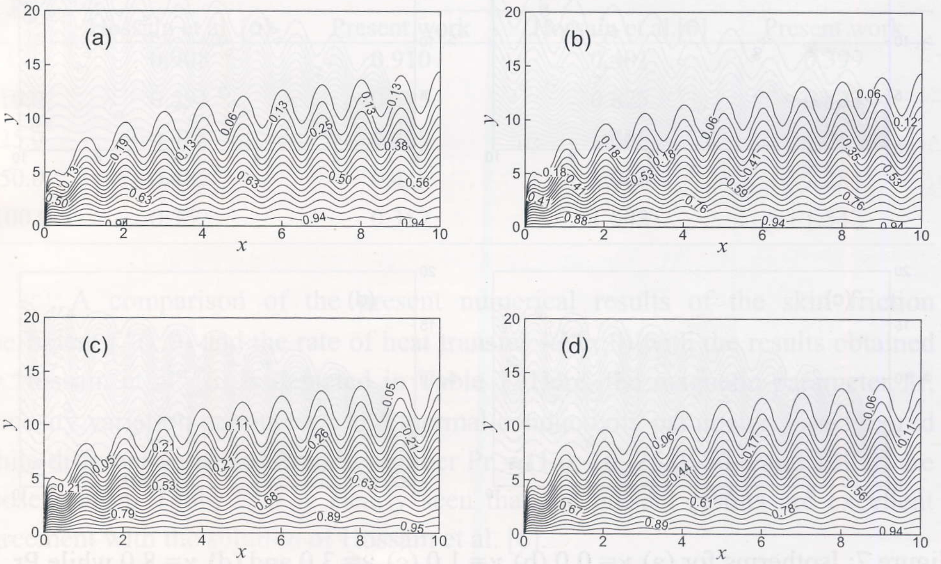

Figure 5: Isotherms for (a) $\varepsilon = 0.0$ (b) $\varepsilon = 0.5$ (c) $\varepsilon = 1.0$ and (d) $\varepsilon = 2.0$ while $\alpha = 0.3$, $M = 0.02$, $\gamma = 3.0$ and Pr = 0.73.

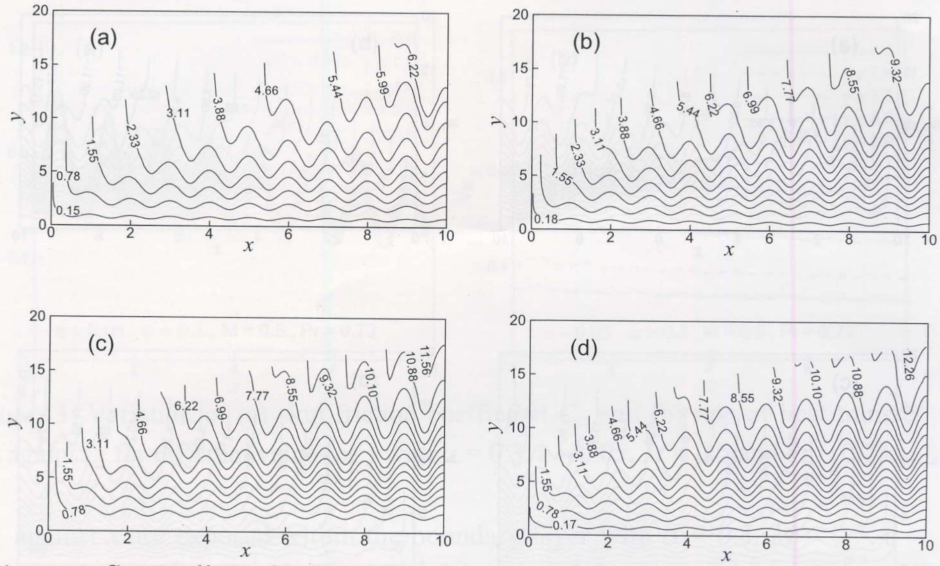

Figure 6: Streamlines for (a) $\gamma = 0.0$ (b) $\gamma = 1.0$ (c) $\gamma = 3.0$ and (d) $\gamma = 8.0$ while Pr = 0.73, $M = 0.5$, $\varepsilon = 0.05$ and $\alpha = 0.3$.

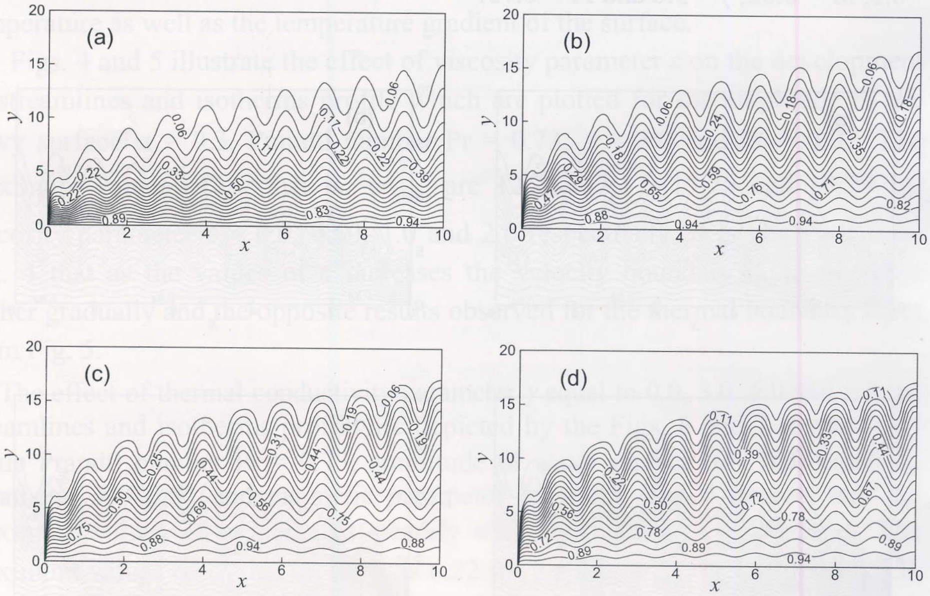

Figure 7: Isotherms for (a) $\gamma = 0.0$ (b) $\gamma = 1.0$ (c) $\gamma = 3.0$ and (d) $\gamma = 8.0$ while Pr = 0.73, $M = 0.5$, $\varepsilon = 0.05$ and $\alpha = 0.3$.

5 CONCLUSION

The effect on MHD free convection flow along a vertical wavy surface with temperature dependent thermal conductivity and viscosity as the inversely proportional to linear function of temperature has been studied in detail. From the present analysis the following conclusions may be drawn:

- The local rate of heat transfer Nu_x, the velocity of the fluid flow increase and skin friction coefficient decreases over the whole boundary layer for increasing values of ε. The thermal boundary layer becomes thinner for increasing value of viscosity variation parameter.
- Increasing values of thermal conductivity parameter γ, the skin friction coefficient C_{fx}, the local rate of heat transfer Nu_x, the velocity and the temperature of the fluid flow significantly increase.

Table 1: Comparison of the present numerical results of skin friction coefficient, $f''(x,0)$ and the heat transfer, $-\theta'(x,0)$ with Hossain et al. [6] for the variation of Prandtl number Pr while $M = 0.0$, $\gamma = 0.0$ and $\varepsilon = 0.0$ with $\alpha = 0.1$.

Pr	$f''(x,0)$		$-\theta'(x,0)$	
	Hossain et al. [6]	Present work	Hossain et al.[6]	Present work
1.0	0.908	0.910	0.401	0.399
10.0	0.591	0.595	0.825	0.823
25.0	0.485	0.489	1.066	1.064
50.0	0.485	0.419	1.066	1.284
100.0	0.352	0.357	1.542	1.542

A comparison of the present numerical results of the skin friction coefficient $f''(x,0)$ and the rate of heat transfer $-\theta'(x,0)$ with the results obtained by Hossain et al. [6] is depicted in Table 1. Here, the magnetic parameter M, viscosity variation parameter ε and thermal conductivity parameter γ are ignored while different values of Prandtl number Pr = (1.0, 10, 25.0, 50.0 and 100.0) are chosen. From Table 1, it is clearly seen that the present results are excellent agreement with the solution of Hossain et al. [6].

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