PERFORMANCE COMPARISON OF BEAMFORMING TECHNIQUES

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ABSTRACT

Smart antenna employing beamforming technique is one of the most promising approaches for combating Multiple Access Interference (MAI). In this paper the performances of two of the most popular criteria for beamforming namely Minimum Mean Square Error (MMSE) and the Maximum Signal to Interference and Noise Ratio (MSINR) have been compared. Their theoretical equivalence with computer simulation have been verified. The effect of the number of samples on the estimation of second order statistics for beamforming is also investigated.

I. INTRODUCTION

Multiple access interference (MAI) is one of the major limiting factors on the capacity of a wireless system. Adaptive antenna array [1] or a smart antenna [1] can be used to combat MAI with the employment of spatial processing. Since the users of a wireless system transmit from different spatial locations, the received signal from each user has a unique spatial signature associated with it. Adaptive antenna arrays can exploit this spatial property of the signal to reduce the MAI by performing beamforming. The beamformer may be a very practical solution to improve the performance of a Code Division Multiple Access (CDMA) system [2], which is designed to operate in co-channel interference. The capacity of a CDMA system can be effectively increased with a small reduction in the co-channel interference levels. This is a marked contrast from Time Division Multiple Access (TDMA) systems which do not benefit as much from a small reduction in interference [3].

An antenna array consists of a set of antenna elements that are spatially distributed at known locations with reference to a common fixed point [4], [5].The antenna elements can be arranged in various geometries. Some of the popular geometrical configurations are Linear, Circular and Planar. In this paper we have employed a uniform linear antenna array. In a linear array, the centers of the elements of the array are aligned along a straight line.

Beamforming [1] is the most common spatial processing technique that an antenna array can utilize. In a wireless system, the desired and the interfering signals originate from different spatial locations. This spatial separation is exploited by a beamformer which can be regarded as a spatial filter separating the desired signal from the interference. The signals from different antenna elements are weighted and summed to *optimize* the quality of the signal. Figure 1 illustrates the idea of a narrowband [1], [4] beamformer. With the proper selection of beamforming criterion, it is possible to *point* the beam towards the direction of the desired user and/or place nulls in the direction of the interferers.

If we have *K* total signals with distinct Angle of Arrival (AOA) impinging on an antenna array consisting of *N* elements, the received signal vector can be written as (1).

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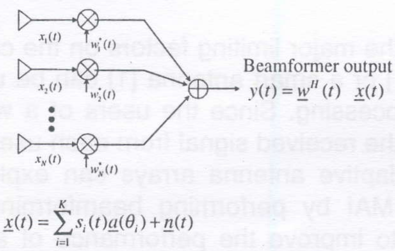
$$\underline{x}(t) = \sum_{i=1}^{K} s_i(t)\underline{a}(\theta_i) + \underline{n}(t) \qquad (1)$$

where $S_i(t)$ is the i^{th} signal with an AOA of θ_i, $\underline{a}(\theta_i)$ is the $N \times 1$ antenna response vector for the AOA of θ_i and $\underline{n}(t)$ is the thermal noise vector. The output of the antenna array is given by

$$y(t) = \underline{w}^H(t)\underline{x}(t) \qquad (2)$$

Here $\underline{w} = [w_1\ w_2 \dots w_N]^T$ is the $N \times 1$ weight vector and H denotes Hermitian transpose. The weight vector is chosen to optimize some beamforming criterion. Popular adaptive beamforming techniques include Minimum Mean Square Error (MMSE) [1], Maximum Signal to Interference and Noise Ratio (MSINR) [1], Maximum Signal to Noise Ratio (MSNR) [6], Constant Modulus (CMA) [3], Maximum Likelihood (ML) [1], etc. In this paper we have investigated the MSINR and MMSE beamforming criteria.



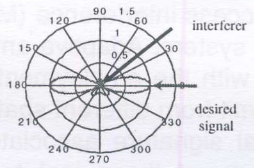


Figure 1(a): Beamformer principle   Figure1(b): Typical array gain pattern

Beamformer that maximizes the Signal to Interference and Noise Ratio (SINR) at the output of the beamformer was proposed by various researchers [7]-[9]. This beamformer termed as the optimal beamformer in fact can be attributed to [10] whose early work by finding the Maximum Likelihood (ML) estimate of the power of the desired signal led to its development. The optimum beamformer is often time termed as the Minimum Variance Distortionless Response (MVDR) Beamformer. In mobile communications literature, the optimal beamformer is often referred to as the optimal combiner. Discussion on the use of the optimal combiner to cancel interferences and to improve the performance of mobile communications systems can be found in [11]–[14]. A beamformer that utilizes the Wiener solution arising from the Minimum Mean Squared Error (MMSE) criterion was proposed in [15]. Further analysis of this technique can be found in [16], [17], [18] and [9].

The MMSE and MSINR beamforming techniques can be shown to be equivalent if it is assumed that the AOAs of the incoming signals as well as their corresponding array response vectors are known [1]. The MMSE beamformer was compared to an MVDR beamformer in [18]. Similar study in a mobile communication environment based on simulation was performed in [19]. However in a practical scenario, the AOAs or the array response vectors are not known. The estimation of the required covariance matrices also has to be performed with a finite number of samples. At the same time many computationally simple algorithms are employed to compute the weights. As a result, the performance of the two beamforming techniques may vary in a practical scenario. In this paper we have compared the performance of the two beamforming techniques when the array response vectors of the desired user and the interferer are not known. Computationally simple Sample Matrix Inversion (SMA)[20] was employed to compute the MMSE weight whereas the MSINR weight was computed by employing the Generalized Power method [21]. The effect of sample size on the estimation of the statistics is also shown in this paper.

Here is how the rest of the paper is organized. Section II provides a brief description of the two beamforming criteria. Section III presents the simulation environment, results and discussion on the results. Section IV concludes this paper.

II. BEAMFORMING ALGORITHM

In this section we discuss the two most common beaming technique employed in cellular communication, namely MMSE and MSINR beamforming criteria.

A. MSINR Beamforming Criteria

MSINR beamforming criterion is intended to maximize the Signal to Interference and Noise Ratio (SINR) at the output of the beamformer. The MSINR beamforming results in a Generalized Eigenvalue problem (GE), [21], [22].

Let us write the received signal vector as

$$\underline{x} = \underline{s} + \underline{u}, \tag{3}$$

where $\underline{s}$ is the desired signal and is $\underline{u}$ the undesired signal which comprises of interference and thermal noise. The Signal to Interference and Noise Ratio (SINR) at the output of the beamformer is [1]

$$SINR_{out} = \frac{\underline{w}^H \underline{\underline{R}}_{ss} \underline{w}}{\underline{w}^H \underline{\underline{R}}_{uu} \underline{w}} \tag{4}$$

Here $\underline{\underline{R}}_{ss} = E(\underline{u}\,\underline{u}^H)$ is the covariance matrix of the desired signal vector $\underline{s}$ and $\underline{R}_{cc} = E(\underline{s}\,\underline{s}^H)$ is the covariance matrix of the interference and noise signal vector $\underline{u}$. In order to find the optimum weight vector that maximizes the output SINR, we have to take the derivative of the right hand side of Equation 4 with respect to $\underline{w}^H$ and set it equal to a null vector. The optimum weight vector that maximizes the SINR is given by the principal eigenvector (the eigenvector corresponding to the maximum eigenvalue) of the following Generalized (or joint) Eigenvalue problem (GE) [22]:

$$\underline{\underline{R}}_{ss} \underline{w}_{MSINR} = \lambda \underline{\underline{R}}_{uu} \underline{w}_{MSINR} \tag{5}$$

The matrix $\underline{R}_{uu}$ can be regarded as an operator modifying the weight vector that one would otherwise obtain from solving a simple eigenvalue problem like $\underline{\underline{R}}_{ss} \underline{w}_{MSNR} = \lambda \underline{w}_{MSNR}$ [22]. The MSINR beamforming can be viewed as a technique that maximizes the SNR for *spatially colored* noise.

If we could assign a single AOA θ_d to the desired signal, the desired signal vector can be written as $\underline{s}(k) = d(k)\underline{a}(\theta_d)$ where $\underline{d}$ is the desired symbol, k is the sample index and $\underline{a}(\theta_d)$ is the array response vector for an AOA of θ_d. As a result, the covariance matrix of the desired signal can be written as

$$\underline{\underline{R}}_{ss} = E\left(\|d\|^2\right)\underline{a}(\theta_d)\underline{a}^H(\theta_d) \tag{6}$$

So from Equation 5, we can write

$$\underline{\underline{R}}_{uu}^{-1}\left\{E\left(\|d\|^2\right)\underline{a}(\theta_d)\underline{a}^H(\theta_d)\underline{w}_{MSINR}\right\} = \lambda_{\max}\underline{w}_{MSINR} \tag{7}$$

By defining $\xi = \dfrac{E\left(\|d\|^2\right)\underline{a}^H(\theta_d)\underline{w}_{MSINR}}{\lambda_{\max}}$, the MSINR weight vector is given by.

$$\underline{w}_{MSINR} = \xi\underline{\underline{R}}_{uu}^{-1}\,\underline{a}(\theta_0) \tag{8}$$

B. MMSE Beamforming Criterion:

The Minimum Mean Squared Error (MMSE) criterion intends to find a weight vector that will minimize the Mean Squared Error (MSE) between the combined signal and some desired (or reference) signal. The error signal can be defined as [1]

$$e(k) = d(k) - \underline{w}^H\underline{x}(k)\,, \tag{9}$$

where d is the reference signal, $\underline{w}$ is the antenna weight vector, $\underline{x}$ is the received signal vector at the antenna array, k is the sample index. Now the MSE is given

$$e(k) = d(k) - \underline{w}^H\underline{x}(k) \tag{10}$$

Here E denotes the ensemble expectation operator. The MSE J is minimized when the gradient vector $\nabla(J)$ is equal to a null vector. By doing that we arrive at the well-known Wiener-Hopf equation [1]

$$\underline{\underline{R}}_{xx}\underline{w}_{MMSE} = \underline{r}_{xd} \tag{11}$$

where $\underline{\underline{R}}_{xx} = E[\underline{x}(k)\underline{x}^H(k)]$ is the covariance matrix of the received signal and $\underline{r}_{xd} = E[\underline{x}(k)d^*(k)]$ is the cross-correlation vector between the received signal vector $\underline{x}$ and the reference signal d. If we premultiply Equation 11 by $\underline{\underline{R}}_{xx}^{-1}$ we get

$$\underline{w}_{MMSE} = \underline{\underline{R}}_{xx}^{-1}\,\underline{r}_{xd} \tag{12}$$

The above solution for MMSE weight is often called the Wiener solution [1]. If the desired signal is uncorrelated with the interference and noise,

$$\underline{\underline{R}}_{xx} = \underline{\underline{R}}_{ss} + \underline{\underline{R}}_{uu} \tag{13}$$

Now if the desired signal had a single AOA θ_d associated with it and the reference signal was the actual desired signal,

$$\underline{\underline{R}}_{ss} = E\left(\|d\|^2\right)\underline{a}(\theta_d)\underline{a}^H(\theta_d)$$

$$\underline{r}_{xd} = E\left(\|d\|^2\right)\underline{a}(\theta_d) \tag{14}$$

By applying *Woodbury's Identity* [1], we get

$$\underline{\underline{R}}_{xx}^{-1} = \left\{\frac{1}{1 + E\left(\|d\|^2\right)\underline{a}^H(\theta_d)\underline{\underline{R}}_{uu}^{-1}\,\underline{a}(\theta_d)}\right\}\underline{\underline{R}}_{uu}^{-1} \tag{15}$$

So the MMSE weight is given by

$$\underline{w}_{MMSE} = \chi \, \underline{\underline{R}}_{uu}^{-1} \, \underline{a}(\theta_0) \, , \tag{16}$$

$$\chi = \left\{ \frac{E\left(\|d\|^2\right)}{1 + E\left(\|d\|^2\right) \underline{a}^H(\theta_d) \underline{\underline{R}}_{uu}^{-1} \underline{a}(\theta_d)} \right\} \tag{17}$$

By comparing Equation 16 with Equation 8, we observe that the MMSE weight vector differs from the MSINR weight vector by a scalar. Since the SINR at the output of the beamformer does not depend on the scalar, *the MMSE weight vector in fact maximizes SINR.*

III. Simulation Results and Analysis

In this section we compare the performance of the MSINR and MMSE beamforming for a simple scenario. The signal transmitted by the desired user is corrupted by two interferers and thermal noise at the receiver which is equipped with a 4 element Uniform Linear Array (ULA) [1] with half wavelength spacing between the omni directional elements.

A. Simulation Environment

The desired user transmits 8ms long slots of QPSK symbols. 7ms of the slot contains actual QPSK symbol that represents the ON time. The remaining 1ms is the OFF period when no signal is transmitted. This OFF period is to facilitate the estimation of interference and noise covariance matrix to perform MSINR based beamforming. At a sampling rate of 25 KHz, there are 25 samples of interference and noise signal. If one sample is used to represent one transmitted symbol, the ON period consists of 175 symbols/samples. Unless explicitly mentioned otherwise, all the statistics is estimated for 25 samples. So for the MMSE beamforming, it is assumed that there are 25 known pilot symbols at the beginning of each slot.

The location of the desired user is at 30^0 with respect to the receiver array broadside. The two interferers are located at 60^0 and -60^0 respectively. The interference is assumed to be wideband zero mean white Gaussian Noise. The only channel impairment is Additive White Gaussian Noised (AWGN). There are no reflectors or scatterers assumed to be present so that there is no angle spread and the position of the transmitters directly translates to AOA.

B. Estimation of Second Order Statistics for Beamforming

The MSINR weight vector is computed by employing the GE given by Equation 5.The GE is solved employing the generalized power [21] algorithm. The required covariance matrices are estimated as an average over a block of data so that

$$\hat{\underline{\underline{R}}}_{xx} = \frac{1}{N_{off}} \sum_{l=0}^{N_{off}-1} \underline{x}(l) \, \underline{x}^H(l)$$

$$\hat{\underline{\underline{R}}}_{uu} = \frac{1}{N_{off}} \sum_{l=N_{on}}^{N_{on}+N_{off}-1} \underline{x}(l) \, \underline{x}^H(l) \tag{18}$$

Here $\underline{x}(l)$ is the received signal vector, N_{on} and N_{off} are the number of samples in the ON and OFF period respectively.

The MMSE weight vector is estimated by applying the Wiener solution given in Equation 12. The estimate of the received signal covariance matrix, $\hat{\underline{\underline{R}}}_{xx}$, is computed according to Equation 18. The cross-correlation vector is estimated as

$$\hat{\underline{r}}_{xd} = \frac{1}{N_{off}} \sum_{l=0}^{N_{off}-1} \underline{x}(l) d^*(l), \qquad (19)$$

where $d^*(l)$ is the conjugate of the known pilot sample. Sample Matrix Inversion technique [20] was employed to compute the inverse of the matrix, $\hat{\underline{\underline{R}}}_{uu}$.

Now, the interfering signals are independent of each other and thermal noise. Each interfering signal can be associated with a discrete AOA. As a result, the actual Interference and Noise covariance matrix is given by

$$\underline{\underline{R}}_{uu} = \sum_{i=0}^{1} \sigma_i^2 \, \underline{a}(\theta_i)\underline{a}^H(\theta_i) + \sigma_n^2 I \qquad (20)$$

Here σ_d^2 is the received signal power of the i^{th} interferer, $\underline{a}(\theta_i)$ is the array response vector of the i^{th} interferer with an AOA of θ_i, σ_n^2 is the variance of the zero mean thermal noise. Since the desired signal has a discrete AOA and it is independent of the interfering signals as well as the noise, the actual received signal covariance matrix is given by

$$\underline{\underline{R}}_{xx} = \underline{\underline{R}}_{uu} + \sigma_d^2 \, \underline{a}(\theta_d)\underline{a}^H(\theta_d) \qquad (21)$$

where σ_d^2 is the power of the desired signal, $\underline{a}(\theta_d)$ is the array response vector of the desired signal with an AOA of θ_d and $\underline{\underline{R}}_{uu}$ is the interference and noise covariance matrix as given by Equation 20.

For an AOA of 30^O, the array response vector of the desired user is given by $[1 \ -0.993-j0.0376 \ 0.9972 + j0.0751 \ -0.9936 \ -j0.11263]^T$. If we estimate the exact interference and noise covariance matrix with Equation 20, we can compute the actual MSINR weight by employing Equation 8. This actual MSINR weight is employed to generate the base-line performance curves. For comparison purposes, we also simulate the performance of a single antenna receiver that has no spatial processing capability.

C. Simulation Results

Figures 3 and 4 show the beam patterns for different levels of interference. The desired user is at 30^0. The interferers are at 60^0 and -60^0 (300^0) respectively. The MSINR and MMSE beam patterns based on the matrix estimates are very similar to the beam pattern of the MSINR weight computed with exact knowledge of the desired signal AOA and the covariance matrix of the interference and noise signal.

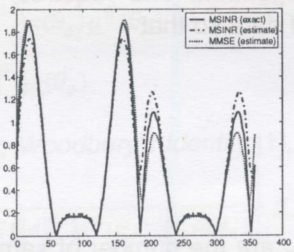


Figure 3: Examples of beam pattern. Both the interferers at 20 dB higher power level

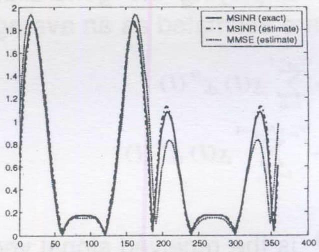


Figure 4: Examples of beam pattern. Both the interferers at 10 dB higher power level.

Figures 5 to 8 show the BER for different interference power level. The single antenna receiver without spatial processing has unacceptable performance. There is a large improvement in performance if beamforming is employed. We can observe that the performance of the MMSE and MSINR beamforming based on the estimated covariance matrices are very similar. Also there is very little degradation compared to the performance of the beamformer that employs the actual MSINR weight.

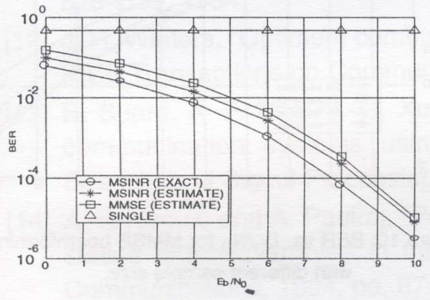


Figure 5: BER vs. E_b/N_0. Both the interferers are being received at 20 dB higher power level

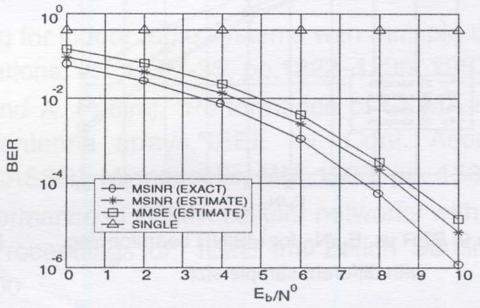


Figure 6: BER vs. E_b/N_0. Both the interferers are being received at 10 dB higher power level

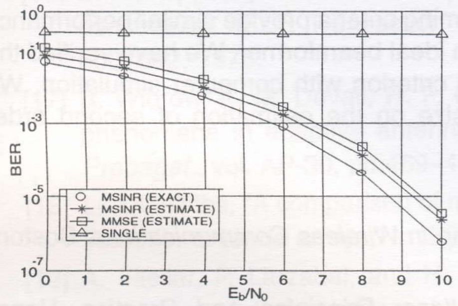


Figure 7: BER vs. E_b/N_0. Both the interferers are being received at equal power level.

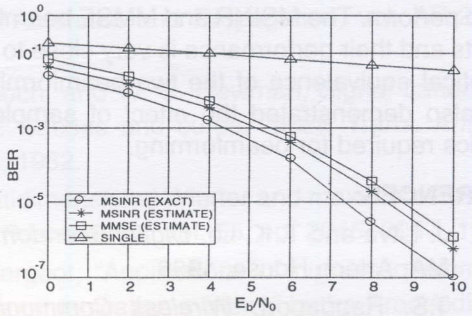


Figure 8: BER vs. E_b/N_0. Both the interferers are being received at 10 dB lower power level.

The next set of simulation results shown in Figures 9 and 10 demonstrate the effect of sample size on the performance of the beamforming. Both the interferers are being received at 20 dB higher power level than the desired signal. We can observe that for both the beamforming criteria, the performance gets better with the increase in sample size. This is expected since the ensemble average provides a better estimate for larger number of samples. We can observe big gain in performance as the sample size increases from 10 to 15 and also from 15 to 20. However the performance does not improve significantly by increasing the sample size from 20 to 25. Therefore there is an optimum number of samples for an accurate estimate of the statistics required for beamforming and for this scenario it can be taken as 20 or 25. The simulations were conducted for static channel condition and in a time varying environment the coherence time [2], defined as the length of time for which the signal retains strong correlation, has to be considered also. The block must contain adequate number of samples for a reliable ensemble average and at the same time it must be small enough so that the channel does not change significantly within the block.

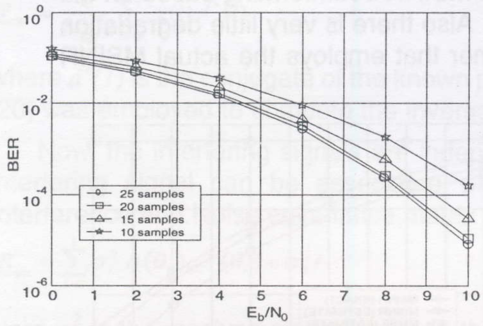


Figure 9: BER vs. E_b/N_0 for MSINR beamforming with different sample size.

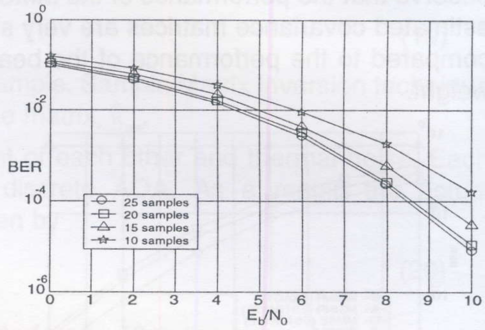


Figure 10: BER vs. E_b/N_0 for MMSE beamforming with different sample size.

IV. Conclusion

We have demonstrated that at the presence of strong interfering signal beamforming allows a wireless receiver to operate successfully whereas a simple receiver will not be able to perform. The MSINR and MMSE beamforming criteria provide similar performance benefits and their performance is very close to an ideal beamformer. We have verified the theoretical equivalence of the two beamforming criterion with computer simulation. We have also demonstrated the effect of sample size on the estimation of second order statistics required for beamforming.

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