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## SIMULATION OF THE PLASTIC FIELD IN THE DEFORMATION ZONE OF SURFACE ROLLING TOOLS UNDER VARIABLE WORK HARDENING CONDITIONS

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#### ABSTRACT:

Longevity of the machine or machine parts depends on the quality of surface layer formed during mechanical working, on the other hand the quality of the surface layer depends on the mechanisms of formation of surface layer. In order to control the quality of the surface layer it is needed to study the formation of surface layer during the mechanical working process. The stresses developed during the work hardening by surface rolling could be studied by slip-line theory or by finite element method. This project employs the slip-line theory in understanding the formation of surface layer.

Development of computer technology and its application in modelling processes avoids costly experimental methods and extensive calculation. In this work, on the basis of theoretical model, computer programming has been developed for the simulation of the plastic field in the deformation zone. Profile of deformation zone has been taken as input of the program. Simulations were done for different profiles. The results were compared with the experimental data. Comparison shows that the proposed simulation method can be used for analysing the plastic field with sufficient accuracy. Developed computer software could be used in order to accelerate the procedure for stress analysis.

*Keywords:* Surface Rolling, Surface layer, slip-line, work hardening, non work-hardening, heart of deformation zone.

#### **1. INTRODUCTION:**

Longevity of the machine plays an important role in the industrial production. It ensures the uninterrupted production line. This longevity of the machine mainly depends upon the exploiting properties of the machine parts, which in term depends upon the quality of the surface layer, which is gradually formed during the finishing stage of mechanical working. So most of the modern research works [1-11] in this field are aimed at finding out the relationship between the exploiting properties of machine parts and the technological condition of the manufacturing methods. But as the exploiting properties of the machine-parts also depend upon the exploiting condition, it is difficult to find out the general relationship.

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Surface roughness, micro hardness and residual stresses in the surface are considered to be the most important parameters of the surface layer in modern techniques [12-15]. Numerous experimental works shows that cold surface rolling is one of the most effective methods of mechanical working which, forms negative (compressive) residual stress in the surface layer in combination with work hardening and smoothness thus increasing the efficiency of the machine part. All this works [16-19] are based on experiments to find empirical relationship between technological parameters and quality of the surface layer without proper study of the mechanisms of formation of the surface layer. It makes difficult to ensure the necessary quality in the surface layer by technological methods. As a result, at present there is no universal quantitative relationship between the parameters of residual stress curve and technological factors. There is no clear conception about the slump of the curve in the surface layer. For this, it is most important for machine building to work out the theoretical model of formation of residual stress for cold surface rolling and methods for calculation of residual stress curve in the designing period of technological process.

One of the techniques which could be adopted for the understanding of creation of residual stresses during the formation of surface layer by surface rolling is to study the plastic field in the deformation zone during loading and elastic field during unloading. In this paper, we tried to study the formation of plastic field during loading. The method used here is the construction of slip line fields and applying the history of loading and unloading for calculating the stresses in the deformation zone. Considering the different boundary conditions, slip line fields can be constructed and the residual stress within the material could be calculated on the basis of variation of mechanical properties of material in the deformation zone.

Development of computer technology and its applications in modelling processes allows us to consider the mechanical properties variation with deformation zone and avoids costly experimental methods and extensive calculations. On the basis of theoretical models, computer programmes have been developed for the simulation of the slip lines in the deformation zone with variable parameters. In this project, we have emphasized into the performance of the surface-rolling tool and tried to build simulation software on the basis of theoretical equations and the empirical formulas.

Slip line method as the fundamental method for engineering theory of work hardening methods in Manufacturing: Most of the mathematical models of theory of plasticity have the common

basis that they consists of solution of differential equations of equilibrium and the conditions of plasticity taking at a time. For this purposes, it is required to ideal the mechanical properties and to use their simplified models.

Different types of simplification of problems [20-27] of plasticity are known now a day. It depends upon the concrete method of operation and possibility of solution of the assigned problems. One of the common approaches is to consider the model of plane deformation. Research works of Smiliansku V.M [28] shows that in the deformation zone during the surface rolling a small zone near to the contact area have plane strain conditions. In this article, plane strain deformation model was considered to study the stress-strain condition in the elastic-plastic deformation zone named as Heart of Deformation (HOD). The stress in the can be found by solving the systems of equations [29].

Where;  $\sigma$  - hydrostatic pressure (normal stress)  $\theta$  - angle between  $\alpha$  lines and the x-axes k yield shear stress





By replacing the above value the following sets of two quasi-linear differential equations will be found as follows:

$$\frac{\partial \sigma}{\partial x} - 2k\cos 2\theta \cdot \frac{\partial \theta}{\partial x} - 2k\sin 2\theta \frac{\partial \theta}{\partial y} = \sin 2\theta \frac{\partial k}{\partial x} - \cos 2\theta \frac{\partial k}{\partial y}$$
$$\frac{\partial \sigma}{\partial y} - 2k\sin 2\theta \cdot \frac{\partial \theta}{\partial x} - 2k\cos 2\theta \frac{\partial \theta}{\partial y} = -\cos 2\theta \frac{\partial k}{\partial x} - \sin 2\theta \frac{\partial k}{\partial y}$$
------(3)

This system equation is of hyperbolic type and its characteristics coincide with slip-lines  $\alpha$  and  $\beta$ .

Along the  $\alpha$  –line and  $\beta$  -line we have respectively:

$$\frac{dy}{dx} = \tan\theta \qquad \qquad d\sigma - 2kd\theta = \frac{\partial k}{\partial x}\partial y - \frac{\partial k}{\partial y}\partial x \qquad (\text{along } \alpha \text{ -line})$$

$$\frac{dy}{dx} = -\cot\theta \qquad \qquad d\sigma + 2kd\theta = -\frac{\partial k}{\partial x}\partial y + \frac{\partial k}{\partial y}\partial x \quad (\text{along }\beta \text{ -line})$$

---- (4)

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This equation can be considered as the modification of Henky's equations for the deformation zone with variable mechanical properties i.e. with variable work hardening conditions. Structure of the equations in no way related with the profile of the HOD. But it is clear from the equations that the  $\alpha$  -line and  $\beta$  -line are perpendicular to each other and have the law of changes of hydrostatic pressure along the lines. This equation explains how hydrostatic pressure ( $\sigma$ ) and the angle of slip line ( $\theta$ ) changes along the slip lines.

#### 2. Construction of Slip-lines:

For the easiness to construct the slip lines the HOD is divided into different subsidiary domains ADE, ABF, FGS, EHT, ABCD, CQUR, FBCQGF, EDCRHE shown in Figure:2. To draw the slip lines in the deformation zone and to analysis the stress-strain condition in the zone it is required to solve the following three problems in the different region of the HOD.



Figure-2: Slip-line field for various boundary value problems.

The three problems are as follows:

- Hertz problem or 1<sup>st</sup> boundary problems in the region under the two intersecting slip lines in ABCD and CQUR.
- Modified 1<sup>st</sup> boundary problem in the region FBCQGF and EDCRHE.
- Problem of koshi or 2<sup>nd</sup> boundary problems in the region ADE, ABF, FGS and EHT.

# 3. Numerical solution of the First boundary value problem in the form of finite differences:

Due to complexity of the equation (4) these problems are in generally solved by numerical methods. This method varies with the technologies methods as the boundary condition changes.

If the positions of two intersections slip lines AB and AC are given (*Figure3*). They may be considered as the boundary of a previously calculated slip line field in some area to the left of AB line and right to AC line. It is required to construct the slip line field to the right AB and AC, assuming the region to be plastic.



Figure3: Slip line fields for first boundary value problem (Regular Domain)

It is known from the theory of plasticity that if the value of  $\sigma$ and  $\theta$  are given one these lines the field is uniquely determined within the curvilinear rectangular ABDC formed by the given slip lines AB and AC. Let AB be a  $\alpha$ -line and AC a  $\beta$ -line. A typical nodal point (i,j) within the field is the intersection of the slip lines through the base points (i,0) and (0,j). Since the value of  $\theta$  is known at all points on AB and AC, the hydrostatic pressures  $\sigma$  and  $\theta$  at any nodal point (i,j) are then obtained by writing the Henky relations in finite difference form.

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[For Non-hardening conditions]

$$\begin{array}{c|c} \sigma(i, j) = \sigma(i, j-1) + \sigma(i-1, j) - \sigma(i-1, j-1) \\ \theta(i, j) = \theta(i, j-1) + \theta(i-1, j) - \theta(i-1, j-1) \end{array} \right| \quad -----(5)$$

For hardening conditions the equation will be modified as follows:  $\sigma(i, j) = \sigma(i-1, j)\{k(i-1, j) + k(i, j)\}\{\theta(i, j) - \theta(i-1, j)\} - A_{\beta}$ 

$$\theta(i, j) = \frac{\{\sigma(i, j-1) - \sigma(i-1, j)\} + \{\theta(i, j-1)\}\{(k(i, j+1) + k(i, j))\}\}}{\{k(i-1, j) + k(i, j)\}\} + A_{\alpha} + A_{\beta}}{[\{k(i, j-1) + 2k(i, j) + k(i-1, j)\}]}$$

Where,

$$A_{\alpha} = \left(\frac{\partial k(i,j)}{\partial y} + \frac{\partial k(i,j-1)}{\partial y}\right) \frac{\{x(i,j) - x(i,j-1)}{2} + \left(\frac{\partial k(i,j)}{\partial x} + \frac{\partial k(i,j-1)}{\partial x}\right) \frac{\{y(i,j) - y(i,j-1)\}}{2}$$

$$A_{\beta} = \left(\frac{\partial k(i,j)}{\partial y} + \frac{\partial k(i-1,j)}{\partial y}\right) \frac{\{x(i,j) - x(i-1,j)}{2} + \left(\frac{\partial k(i,j)}{\partial x} + \frac{\partial k(i-1,j)}{\partial x}\right) \frac{\{y(i,j) - y(i-1,j)\}}{2}$$

$$\frac{\partial k}{\partial x} = \left(\frac{k(x + \Delta x, y) - k(x - \Delta x, y)}{2\Delta x}\right) - -----(6)$$

$$\frac{\partial k}{\partial y} = \left(\frac{k(x, y + \Delta y) - k(x, y - \Delta y)}{2\Delta y}\right)$$

The coordinate point for both the cases will be calculated as follows:  $x(i, j) = x(i-1, j-1) + [\{(x(i, j-1) - x(i-1, j)\} \cos\{\theta((i, j-1) + \theta(i-1, j)\}] + [\{(y(i, j-1) - y(i-1, j)\} \sin\{\theta((i, j-1) + \theta(i-1, j)\}]$ 

-----(7)

$$y(i, j) = y(i-1, j-1) + [\{(y(i, j-1) - y(i-1, j)\} \cos\{\theta((i, j-1) + \theta(i-1, j)\}] + [\{(x(i, j-1) - x(i-1, j)\} \sin\{\theta((i, j-1) + \theta(i-1, j)\}]$$

Solution of the modified 1<sup>st</sup> boundary problems in the finite difference forms.

If the radius of curvature of one of the given slip lines, say AC, is allowed to vanish, while the change in angle between A and C is held constant (*Figure 4*), we obtain a centred fan ABD defined by slip line AB and the angular difference at A. All  $\alpha$ -line pass through A, which is a singularity of stress since the hydrostatic pressure at this point has different value for each slip line and the  $\alpha$ -lines have the same radius of curvature at A, where  $\beta$ -line is of zero radius.

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The values of  $\sigma$  and  $\theta$  at a typical nodal point (i,j) are found as before, with p(i-1,j) and  $\varphi(i-1,j)$  referring to the appropriate  $\alpha$ -lines at A.



Figure 4 : Slip line fields for first boundary value problem (Modified domain)

4. Solution of the Second boundary value problem in the form of finite differences.





Consider the situation where the normal and tangential stresses are prescribed along a given curve AC and it is required to construct the slip line field below AC on the assumption that the material is plastic. In this case the field is uniquely defined within the curvilinear triangle ABC bounded by the slip lines through A and C (Figure 5). At each point on AC, the normal stress component acting parallel to the boundary can have two different values satisfying the yield criterion. Physical considerations will indicate the correct value of the stress, and hence specify the values p and  $\varphi$  along the boundary. It is not generally possible to chose and equiangular net unless AC is a contour of constant p or of constant  $\varphi$ . The former occurs when the boundary is acted upon by a uniform normal stresses and zero tangential stresses. The latter occurs when a constant frictional stress acts along the boundary with an arbitrary distribution of normal stress.

Let the given AC be divided into an arbitrary number of small segments by the points (1,1), (2,2), (3,3), etc. A typical nodal point (i,j) in the field is defined by the intersection of the slip lines passing through (i,i) and (j,j) on the boundary. Assuming AB to be a  $\alpha$ -line, by considering the Henky's equation in the form of finite differences we have:

[For Non-hardening conditions]

$$\sigma(i, j) - \sigma(j, j) = 2k[\theta(j, j) - \theta(i, j)]$$
  

$$\sigma(i, j) - \sigma(i, i) = 2k[\theta(i, j) - \theta(i, i)]$$
(8)

For hardening conditions the equation will be modified as follows:

$$\sigma(i, j) = \sigma(i - 1, j) + \{k(i - 1, j) + k(i, j)\}\{\theta(i, j) - \theta(i - 1, j)\} + hb$$

$$[\{\theta(i, j - 1) - \theta(i - 1, j)\} + \{\theta(i, j - 1)(k(i, j + 1) + k(i, j))\}\}$$

$$\theta(i, j) = \frac{+\{\theta(i - 1, j)\{\{k(i - 1, j) + k(i, j)\}\} + \gamma a - hb}{[\{k(i, j - 1) + 2k(i, j) + k(i - 1, j)\}]}$$

Where,

$$hb = \frac{1}{2} \left[ \left( \frac{\partial k}{\partial x} \right)_{i-1,j} + \left( \frac{\partial k}{\partial x} \right)_{i,j} \right] \{ y(i,j) - y(i-1,j) \} - \frac{1}{2} \left[ \left( \frac{\partial k}{\partial y} \right)_{i-1,j} + \left( \frac{\partial k}{\partial y} \right)_{i,j} \right] \{ x(i,j) - x(i-1,j) \}$$

$$\gamma a = -\frac{1}{2} \left[ \left( \frac{\partial k}{\partial x} \right)_{i,j-1} + \left( \frac{\partial k}{\partial x} \right)_{i,j} \right] \{ y(i,j) - y(i,j-1) \} + \frac{1}{2} \left[ \left( \frac{\partial k}{\partial y} \right)_{i,j-1} + \left( \frac{\partial k}{\partial y} \right)_{i,j} \right] \{ x(i,j) - x(i,j-1) \}$$

The new coordinate point for both the second boundary cases will be calculated as follows:

$$x(i, j) = \frac{y(i-1, j) - y(i, j+1) + x(i, j+1) \tan \theta(i, j+1) + x(i, j+1) \cot \theta(i-1, j)}{\tan \theta(i, j+1) + \cot \theta(i-1, j)}$$

$$(10)$$

$$y(i, j) = y(i, j+1) + [\{(x(i, j) + x(i, j+1) \tan \theta(i, j+1)\}]$$

#### 5. Mathematical Modelling:

For creating the theoretical model of the above problems numerical methods were employed due to the complexicity of the equations. For that purposes a computer simulation software is proposed. This software enables us to simulate the plastic field in the HOD taking its profile as input as shown in Figure 6.

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For calculating different elements of the profile following inputs and related formulas are used:

## Input data:

 $\mathbf{R} \rightarrow$  Radius of the Ball

- $d \rightarrow$  Length of the leading side contact.
- $d1 \rightarrow$  Length of the trailing side contact.
- $L \rightarrow$  distance from the center of the ball to the point on the surface of the material where there is no influence of surface rolling.
- $hg \rightarrow$  maximum depth measured from the unaffected surface of the material on the leading side.
- $\Delta \rightarrow$  maximum depth measure from the contact of material with the ball on the trailing side.

## **Related data:**

 $hp = R - \sqrt{R^2 - d^2} \rightarrow maximum depth measure from the contact of$ 

material with the ball on the leading side.

 $2b \rightarrow$  projected width of contact the ball makes with the material.  $2a \rightarrow$  projected length of contact the ball makes with the material.

hr = 
$$z \sqrt{\frac{P}{2 \times yield}} - 1.42ab \rightarrow$$
 depth of hardening (empirical formula).

Where, 
$$z = 1 - \frac{1}{2} \left( 1 - \frac{b}{a} \right)^4$$

P = -5.14\*kmax  $\rightarrow$  Hydrostatic pressure. kmax  $\rightarrow$  shear strength of the material.

$$\theta_2 = \cos^{-1}\left(\frac{R-\Delta}{R}\right) \Rightarrow \text{ angle subtended by contact of material on the}$$

trailing side with the ball at the center of the ball.

 $\theta_1 = \cos^{-1} \left( \frac{R - h_p}{R} \right) \Rightarrow \text{ angle subtended by contact of material on the}$ 

leading side with the ball at the center of the ball.

If y<hr then,  $k(x,y) = k_{org}$ 

If y < L(y),  $k(x,y) = k_{org}$ 

If x < 0, k(x,y) = k(y)

For other cases,  $k(x,y) = k \text{ orig } + (k(y) - korg)e^{-Cdelta\left(\frac{xo}{L(y)}\right)}$  $k(y) = k \text{ orig } + (k \max - korg)e^{-Cdelta \times chi^2}$ 





Figure 6: Formation of surface layer during work hardening by surface rolling 6. Simulation software:

Flow chart of the software is shown in Figure:8. The software consists of the following modules:

**Module No: 01** Calculation of initial boundary values: Calculation of the coordinates of the characteristics points, the hydrostatic pressure and the direction of the slip line at that point. For the benefits of easy calculation boundary profile of the HOD always divided into several points. These points are indexed shown in Figure7:

$$d\theta_2 = \frac{\langle BOA}{n_2} \quad d\theta_1 = \frac{\langle BOC}{n_1} \quad d\theta_3 = \frac{\frac{11}{2} - \langle BOC}{n_2} \quad d\theta_3 = \frac{\varphi - \langle BOA}{n_3} \quad ---(11)$$

Where n1,n3 are the number of divisions in the trailing side and n2, n4 are the number of divisions in the leading side. Indexing is done as per following Figure-7



Figure 7: Index of changing points

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Each point are indexed by two numbers. First number is the index of the  $\alpha$  line and second number is the index of  $\beta$  line. Coordinate of the points (leading side) are calculated by the formula:

$$x(i, j) = R \sin[(i - k_3)d\theta_1]$$
  

$$y(i, j) = -R + R \cos[(i - k_3)d\theta_1]$$
  

$$\sigma(i, j) = k(i, j)$$
  

$$\theta(i, j) = -\frac{\Pi}{4} + (i - k_3)d\theta_1$$
------(12)

Coordinate of the points (trailing side) are calculated y the formula:

$$x(i, j) = R \sin[(i - k_3)d\theta_2]$$
  

$$y(i, j) = -R + R \cos[(i - k_3)d\theta_2]$$
  

$$\sigma(i, j) = k(i, j)$$
  

$$\theta(i, j) = -\frac{\prod}{A} + (i - k_3)d\theta_2$$

In the point A,  $\theta(i, j) = \theta_{k4k4} + (j - k4)d\theta_3$ 

$$\sigma(i, j) = \sigma_{k4k4} (1 - \frac{j - k_4}{n_2})$$
 -----(14)

In the point C,  $\theta(i, j) = \theta_{k2k2} + (j - k2)d\theta_4$ 

$$\sigma(i, j) = \sigma_{k_{2k_{2}}}(1 + \frac{i - k_{2}}{n_{2}})$$
(15)

Where, k1 = n1 + n3 + 2, k2 = (2n1 + 1) + n3k3 = 2n1 + n2 + n3 k4 = 2(k3 + 1) + n3k5 = k2 - 1 k6 = k3 + 1

**Module No:02(ABGF-trailing side:** referred to Figure:3): Calculate the value of  $\sigma$  and  $\theta$  of nodal point by using the formula (6), (7), (9) & (10). For the point just below the trailing side formula for second boundary problem is used. For other points first boundary problem formulas are used.

**Module No:03(ADHE :** leading side: referred to Figure:3): Calculate the value of p and  $\varphi$  of nodal point by using the formula (6), (7), (9) & (10). For the point just below the leading side formula for second boundary problem is used. For other points first boundary problem formulas are used.

Module No:04(ABGQURHD: referred to Figure:3): For the point just below the leading and trailing side formulas for first boundary problem are used.

**Module No:05(FGS :** referred to Figure:3): Zone adjacent to the free surface at the trailing side. For the point just below the free zone formula for second boundary problem is used. For other points first boundary problem formulas are used.

**Module No:06(EHT: referred to Figure:3):** Zone adjacent to the free surface at the leading side. For the point just below the free zone formula for second boundary problem is used. For other points first boundary problem formulas are used.

**Programme Code:** The programming software used for the coding of the simulation programme was Turbo C++. The codes were written for both Non-hardening and work hardening simulations into separate files; this was due to the lack memory handling capacity in Turbo C++. The codes of the simulation programme for work hardening and Non-hardening are investigated. The various parameters of all the nodes are tabulated in a text file.

## 7. Simulation of the plastic field:

For the simulation the following sample data tabulated in Table: 1 are used in the simulation software as input data for both non-hardening and work hardening condition during surface rolling.

#### Radius of the ball diameter: 4mm

Tests	P (N/mm <sup>2</sup> )	d (mm)	d1 (mm)	Delta (mm)	L(mm)	h <sub>b</sub> (m m)
1	250	0.325	0.206	0.007	1.299	0.0085
2	500	0.450	0.260	0.014	1.659	0.0175
3	1000	0.625	0.345	0.026	2.123	0.0366
4	1600	0.800	0.419	0.041	2.5546	0.064
5	1800	0.850	0.440	0.046	2.673	0.0738

#### Table 1: Sample data used for the simulation

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Figure 8: Summarized programme flowchart



Figure 9: Simulation for Test 1. (a) Non hardening. (b) Work Hardening



Figure 10: Simulation for Test 2. (a) Non hardening. (b) Work HardeningJournal of Engineering and Technology Vol. 5 No. 1, 200614





Figure 13: Simulation for Test 5. (a) Non hardening. (b) Work Hardening

## 8. Verification and validation of the software:

Verification and validation (V&V) of the computational simulations are the primary methods for building and quantifying the confidence. For this purposes, different theoretical and experimental studies are performed. Guidelines proposed in [30] are followed.

To verify the stability of the solution theoretical experiment was done for different conditions changing the dimensions, forms of the profile and distribution of k-values. These experiments show that the calculated results are stable and changes as theory predicts as shown in Fig:09-13. The inputs for the simulations are tabulated in Table:1.

For the validation purposes, experimental data available in [31] is compared with the simulated results as shown in Fig: 14-17. The comparison shows that the calculated results are in good agreement with the experimental results with sufficient accuracy.

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Fig:14: Distribution of plastic stresses  $(\sigma_{X})$  along the Heart of Deformation.





Fig:16: Distribution of plastic stresses  $(\sigma_z)$  along the Heart of Deformation.

Fig:17: Distribution of plastic stresses in  $(\tau_{xy})$  along the Heart of Deformation.

#### 9. CONCLUSIONS:

A theoretical model for the simulation of plastic field using slip-line method was proposed. Computer software on this theoretical model was prepared. This model can be used both for non-hardening and work hardening method. Theoretical experiments show that this model give a stable results and coincides with the theoretical predictions. Comparison with experimental data shows that the theoretical results are valid for the allowable technological parameters. This software will create a good opportunity for the prediction of the properties of the surface layer during design stage.

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