Natural Convection in a Vee-corrugated Square Enclosure with Discrete Heating from below

Goutam Saha*, Sumon Saha**, Mohammad Ali** and Md. Quamrul Islam**

ABSTRACT

A study on steady-state natural convection heat transfer and fluid flow in a square enclosure with Vee-corrugated vertical walls has been carried out using finite element based adapting meshing technique. The present study is based on a configuration where a constant flux heat source is discretely embedded at the bottom wall. The length of the heat source is 20% of the total length of the bottom wall. The non-heated parts of the bottom wall and entire upper wall are considered adiabatic. The pressure-velocity form of the Navier–Stokes equations and energy equation are used to represent the mass, momentum, and energy conservations of the fluid medium in the cavity. The finite element formulations of the dimensionless governing equations with the associated boundary conditions are solved by a nonlinear coupled solution algorithm using a six nodded triangular element discretization scheme for all the field variables. The Grashof number based on the enclosure height is varied from 10^3 to 10^6, corrugated frequency is varied from 0.5 to 2 and Prandtl number is taken as 0.71. In this work, we examine the effect of corrugation frequency and the buoyancy force parameter on the flow and heat transfer characteristics. Results are presented in the form of streamlines and isotherm plots. The investigation shows that the average Nusselt number is maximum for low corrugation frequency but the reverse trend is found for high corrugation frequency.

Keywords : Natural convection, Square enclosure, Constant heat flux, Vee-corrugated wall.

NOMENCLATURE

g	Gravitational acceleration
Gr	Grashof number
W	Width or Height of the enclosure
k	Thermal conductivity of air
L	Length of the heat source

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P	Dimensionless local pressure
Pr	Prandtl number
q''	Heat flux at the source
U	Dimensionless velocity component in X-direction
V	Dimensionless velocity component in Y-direction
u	Dimensional velocity component in x-direction
v	Dimensional velocity component in y-direction
X, Y	Non-dimensional coordinates
x, y	Dimensional coordinates

(Greek symbols)

α	Thermal diffusivity
ρ	Fluid density
β	Thermal expansion coefficient
θ	Dimensionless temperature
υ	Kinematic viscosity
φ	Any dependent variables

1 INTRODUCTION

Natural convection is observed as a result of the motion of the fluid due to density changes arising from heating process. The movement of the fluid in free convection results from the buoyancy forces imposed on the fluid when its density in the proximity of the heat transfer surface is described as a result of thermal expansion of the fluid in a non-uniform temperature distribution. Convection heat transfer is dependent on the movement of the fluid and the development of the flow of the fluid is also influenced by the shape of the heat transfer surfaces. Both numerical and experimental methods have been used to obtain the solution of heat transfer and fluid flow problems. Though experimental methods are more realistic, they are costly and time consuming due to necessary of expensive prototypes and instrumentation. On the otherhand, numerical methods can offer considerable savings in design time and costs.

The problem of convective heat transfer in an enclosure has been studied extensively because of the wide application of such process. Ostrach [1] provided a comprehensive review article and extensive bibliography on natural convection in cavities. Other articles on the topic published are Valencia and Frederick [2], Selamet et al. [3], Hasnaoui et al. [4], Papanicolaou and Gopalakrishna [5], Sundstrom and Kimura [6], Hsu and Chen [7], Elsherbiny et al. [8], and Nguyen and Prudhomme [9], among others, who investigated natural convection in rectangular enclosures under various configurations and orientations. Anderson and Lauiat [10] studied the natural convection in a vertical square cavity heated from bottom and cooled from one side. Convection in a similar configuration, where the bottom wall of the rectangular cavity was partially heated with cooling from one side, was studied by November and Nansteel [11]. It was reported that the heated fluid layer near the bottom wall remains attached up to the turning corner. Ganzarolli and Milanez [12] performed numerical study of steady natural convection in rectangular enclosures heated from below and symmetrically cooled from the sides. The size of the cavity was varied from square to shallow where the cavity width was varied from 1-10 times of the height. The heat source, which spanned the entire bottom wall, was either isothermal or at constant heat flux condition. Aydin and Yang [13] numerically investigated the natural convection of air in a vertical square cavity with localized isothermal heating from below and symmetrical cooling from sidewalls. The top wall as well as non-heated parts of the bottom wall was considered adiabatic. The length of the symmetrically placed isothermal heat source at the bottom was varied. They did not investigate the effect of aspect ratio and inclination of the cavity on the heat transfer process.

Several investigations have been carried out on natural convection heat transfer and fluid flow with corrugated surfaces. Chinnappa [14] carried out an experimental investigation on natural convection heat transfer from a horizontal lower hot Vee-corrugated plate to an upper cold flat plate. He took data for a range of Grashof numbers from 10^4 to 10^6. The author noticed a change in the flow pattern at $Gr = 8 \times 10^4$ which he concluded as a transition point from laminar to turbulent flow. Randall [15] studied local and average heat transfer coefficients for natural convection between a Vee-corrugated plate and a parallel flat plate to find the temperature distribution in the enclosed air space. From this temperature distribution they used the wall temperature gradient to estimate the local heat transfer coefficient. Using control volume based finite element method; Ali [16] also investigated natural convection in a differentially heated enclosure with Vee-corrugated vertical walls. Zhong et al. [17] carried out a finite-difference study to determine the effects of variable properties on the temperature and velocity fields and the heat transfer rate in a differentially heated, two dimensional square enclosures. Nayak et al. [18] considered the problem of free and forced convection in a fully developed laminar steady flow through vertical ducts under the conditions of constant heat flux and uniform peripheral wall temperature. Chenoweth et al. [19] obtained steady-state, two dimensional results from the transient Navier-Stokes equations given for laminar convective motion of a gas in an enclosed vertical slot with large horizontal temperature differences.

The physical model considered here is shown in Fig.1, along with the important geometric parameters. It consists of a square enclosure of dimensions, $W \times W$, whose Vee-corrugated vertical walls are kept at a constant temperature, θ_c. The bottom wall has an embedded symmetrical heat source with constant heat flux, q and length L has been fixed at 20% of the length of the enclosure. The remaining parts of the bottom wall and the entire upper wall are kept adiabatic. The natural convection parameter, Grashof number is varied from 10^3 to 10^6. The corrugation amplitude has been fixed at 10% of the enclosure height. The objective of this work consists in studying the effect of corrugation frequencies for natural convection in a square enclosure, with constant flux heating from below. Also the results are presented in terms of the variation of the average Nusselt number and maximum temperature at the heat source surface.

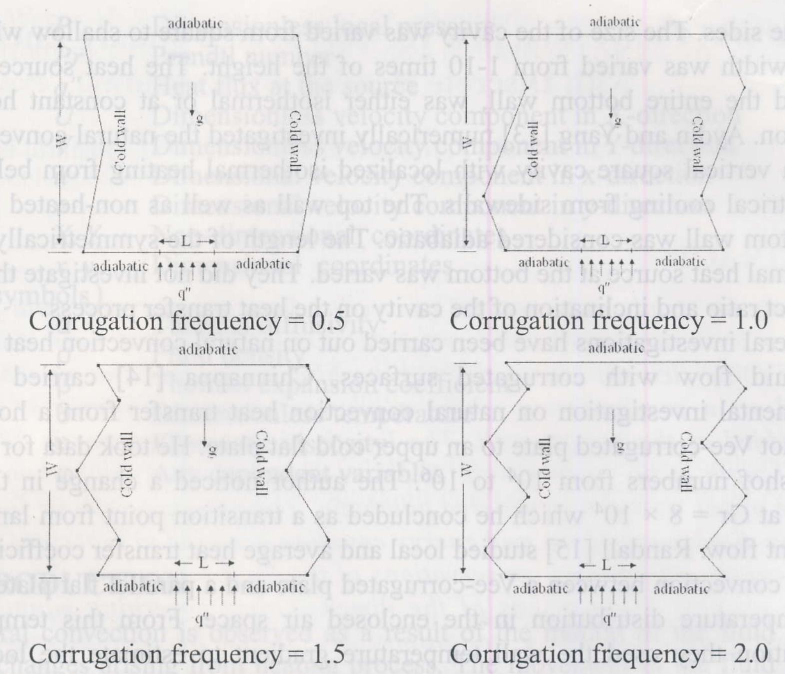

Figure 1: Schematic diagram of the calculation domain

2 MATHEMATICAL MODEL

Natural convection is governed by the differential equations expressing conservation of mass, momentum and energy. The present flow is considered steady, laminar, incompressible and two-dimensional. The viscous dissipation term in the energy equation is neglected. The Boussinesq approximation is invoked for the fluid properties to relate density changes to temperature changes, and to couple in this way the temperature field to the flow field. Then the governing equations for steady natural convection can be expressed in the dimensionless form as:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Gr\theta \tag{3}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\Pr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

where X and Y are the coordinates varying along horizontal and vertical directions, respectively, U and V are, the velocity components in the X and Y directions respectively, θ is the temperature and P is the pressure.
The non-dimensional numbers seen in the above, Gr and Pr are the Grashof number and Prandtl number, respectively, and they defined as:

$$\mathrm{Gr} = \frac{g\beta \Delta t\, W^3}{2} \quad \text{and} \quad \mathrm{Pr} = \frac{\upsilon}{\alpha} \tag{5}$$

The dimensionless parameters in the equations above are defined as follow:

$$X = \frac{x}{W},\ Y = \frac{y}{W},\ U = \frac{uW}{\upsilon},\ V = \frac{vW}{\upsilon},$$
$$P = \frac{pW^2}{\rho \upsilon^2},\ \theta = \frac{T - T_c}{\Delta t},\ \Delta t = \frac{q''W}{k} \tag{6}$$

where ρ, β, υ, and g are the fluid density, coefficient of volumetric expansion, kinematic viscosity, thermal diffusivity, and gravitational acceleration, respectively. The boundary conditions for the present problem are specified as follows:

$$\begin{aligned} &\text{All walls: } U = V = 0 \\ &\text{Top wall: } \frac{\partial \theta}{\partial Y} = 0 \\ &\text{Right and left wall: } \theta = 0 \end{aligned} \tag{7}$$

$$\text{Bottom wall: } \frac{\partial \theta}{\partial Y} = \begin{cases} 0, & for\, 0 < X < 0.4 \\ -1, & for\ 0.4 \le X \le 0.6 \\ 0, & for\, 0.6 < X < 1 \end{cases}$$

Also the dimensionless heat flux at the bottom wall is 1/Pr. The local Nusselt number and the average Nusselt number can be obtained respectively as

$$Nu_x = \frac{1}{\theta_s(X)}$$
$$Nu = \frac{W}{L}\int_0^{L/W} \frac{1}{\theta_s(X)}\, dX \tag{8}$$

where $\theta_s(X)$ is the local dimensionless temperature.

3 NUMERICAL PROCEDURE

The velocity and thermal energy equations (1)-(4) result in a set of non-linear coupled equations for which an iterative scheme is adopted. To ensure convergence of the numerical algorithm the following criteria is applied to all dependent variables over the solution domain

$$\sum\left|\phi_{ij}^{m}-\phi_{ij}^{m-1}\right|\leq 10^{-5} \tag{9}$$

where ϕ represents a dependent variable U, V, P, and θ, the indexes i, j indicate a grid point, and the index m is the current iteration at the grid level. The six node triangular element is used in this paper for the development of the finite element equations. All six nodes are associated with velocities as well as temperature, only the corner nodes are associated with pressure. This means that a lower order polynomial is chosen for pressure and which is satisfied through continuity equation. The iteration process is terminated if the percentage of the overall change compared to the previous iteration is less than the specified value.

4 RESULTS AND DISCUSSION

In this investigation, streamline and isotherm distributions inside the cavity and average Nusselt number distribution at the heated surface have been examined and discussed for corrugation frequency (C.F.) = 0, 0.5, 1, 1.5 and 2, the Grashof number, Gr was varied from 10^3 to 10^6, and the working fluid is chosen as air with Prandtl number, Pr = 0.71. The normalized length of the constant flux heat source at the bottom wall was fixed at 0.2. The numerical procedure used to solve the governing equations for the present work is the finite element based adapting meshing technique. The application of this technique is well documented in [20].

In order to obtain grid independent solution, a grid refinement study is performed for a square enclosure at Gr = 10^3, C.F. = 1.0. Fig.2 shows the convergence of the average Nusselt number, Nu, at the heated surface with grid refinement. It is observed that grid independence is achieved with 5958 elements where there is insignificant change in Nu, with further increase of mesh elements.

Table 1: Comparison of the results of average Nusselt number for validation at C.F. = 0 and $L/W = 0.2$

Gr	Nu		Error (%)
	Present	Sharif et al. [21]	
10^3	5.9393163	5.926608	0.21
10^4	5.9538746	5.946352	0.13
10^5	7.116853	7.124055	0.10
10^6	11.226486	11.34151	1.01

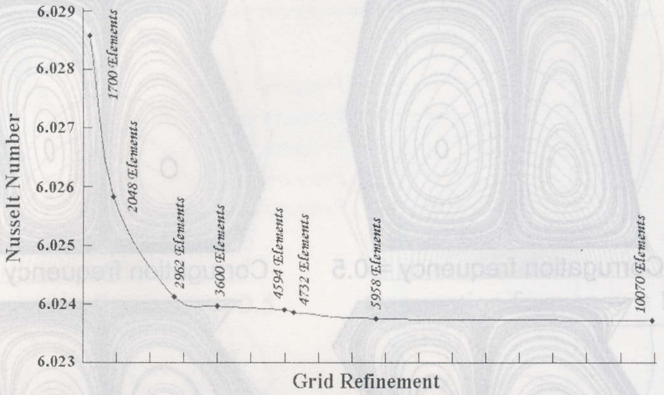

Figure 2: Convergence of average Nusselt number with grid refinement for $Gr = 10^3$ and C.F. = 1.0

The present numerical approach was verified against the results published by Sharif and Mohammad [21] for natural convection heat transfer in square enclosure with straight vertical walls (C.F. = 0) for different Grashof number, Gr as shown in Table 1 and Table 2. It is seen in this comparison that both average Nusselt number and maximum surface temperature are in good agreement. The agreement is found to be excellent with a maximum discrepancy of about 1%, which validates the present computations indirectly.

Table 2: Comparison of the results of maximum temperature for validation at C.F. = 0 and $L/W = 0.2$

Gr	θ_{max}		Error (%)
	Present	Sharif et al. [21]	
10^3	0.181827	0.18194	0.06
10^4	0.181777	0.18176	0.01
10^5	0.157058	0.15682	0.15
10^6	0.109866	0.10920	0.61

The streamlines and isotherms for various corrugation frequencies are shown in Fig.3, Fig.4, Fig.5 and Fig.6 for Grashof number of 10^4 and 10^6 as representative cases. In each case the flow rises along the vertical symmetry axis and gets blocked at the top adiabatic wall, which turns the flow horizontally towards the isothermal cold walls. The flow then descends downwards along the sidewalls and turns back horizontally to the central region after hitting the bottom wall. The isotherm plots are also symmetrical about the vertical mid plane and concentrated towards the hot surface indicating the presence of a large temperature gradient there. For lower Gr (10^3 and 10^4) the convection intensity in the cavity is very weak as evident from the stream function values which

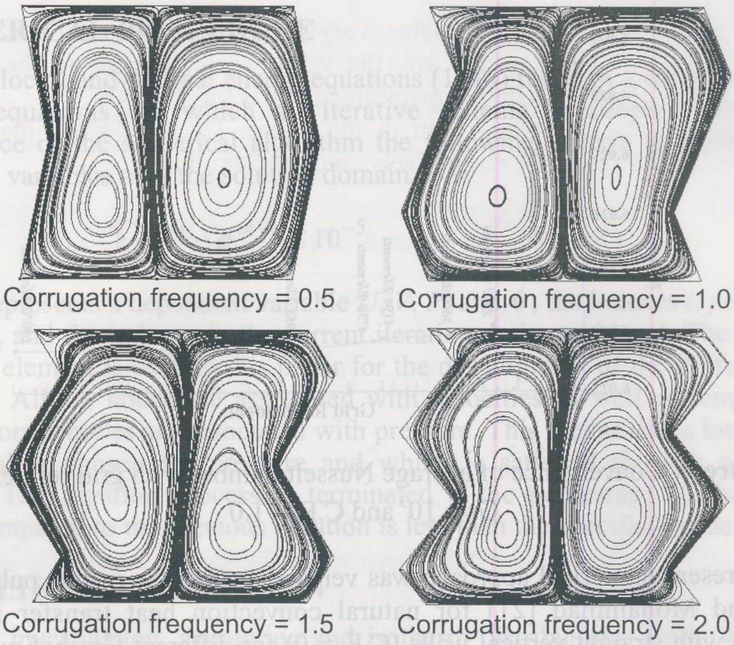

Figure 3: Streamlines in the enclosure for Gr = 10^4

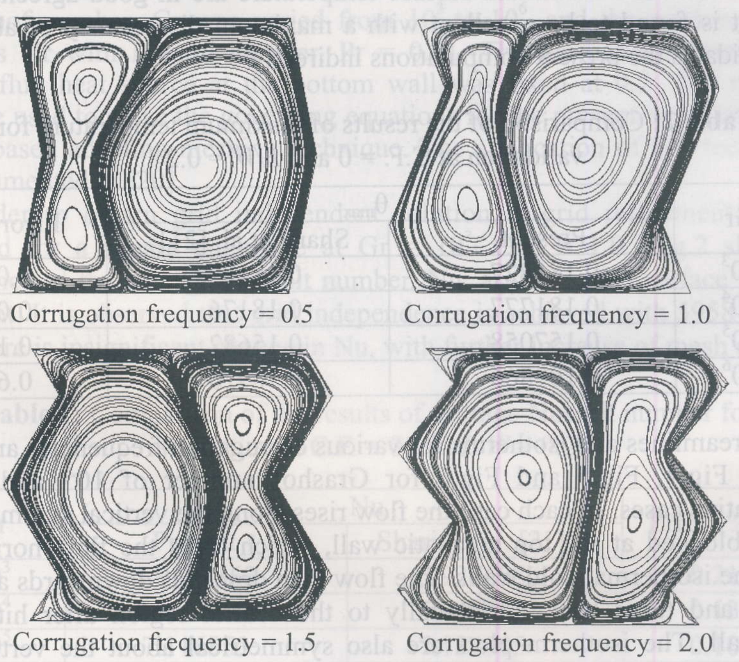

Figure 4: Streamlines in the enclosure for Gr = 10^6

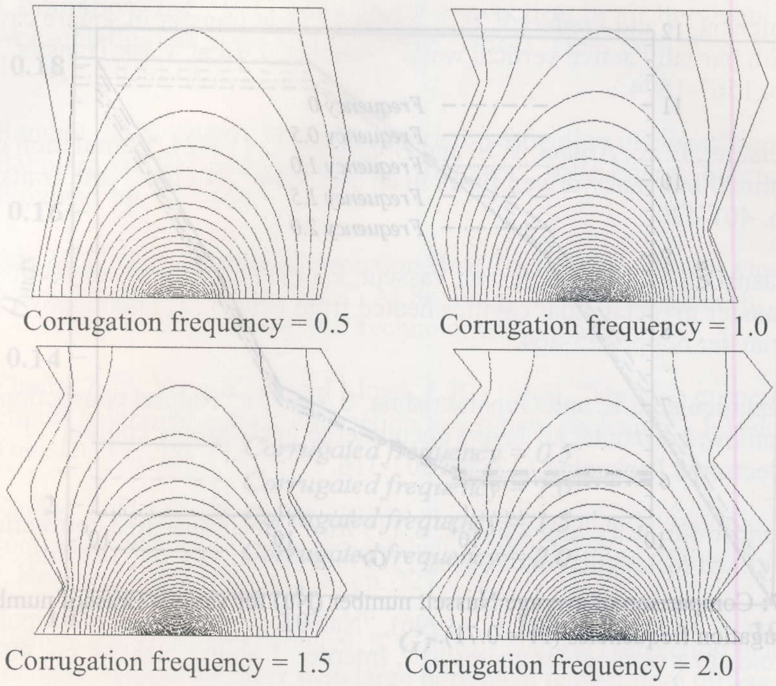

Figure 5: Isotherms in the enclosure for Gr = 10^4

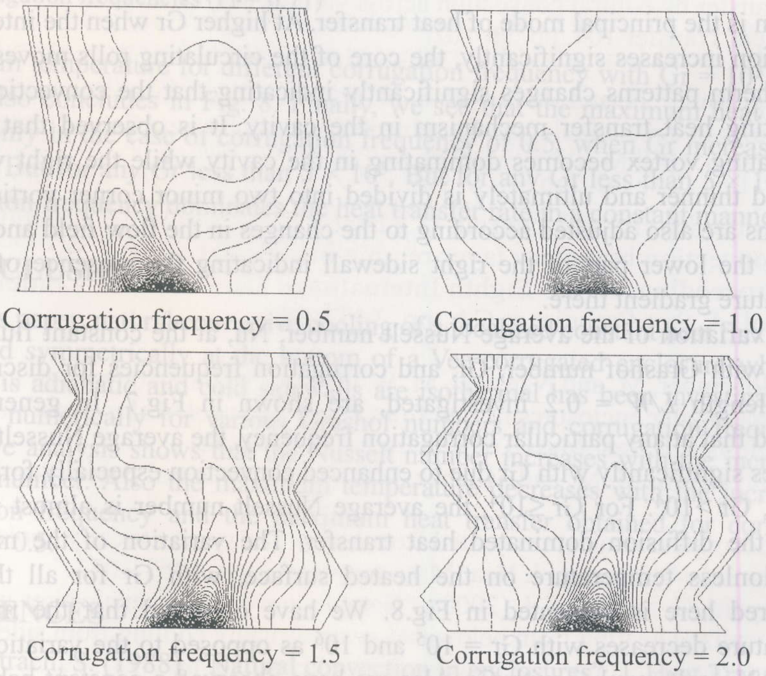

Figure 6: Isotherms in the enclosure for Gr = 10^6

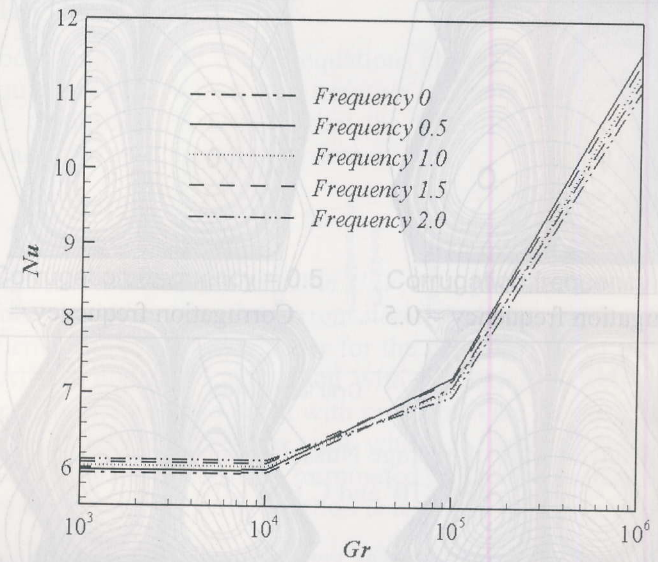

Figure 7: Comparison of average Nusselt number (Nu) for various Grashof numbers (Gr) and corrugation frequencies (Pr = 0.71).

are at least an order of magnitude smaller that those for Gr (10^5 and 10^6). Thus viscous forces are more dominant than the buoyancy forces at lower Gr and diffusion is the principal mode of heat transfer. At higher Gr when the intensity of convection increases significantly, the core of the circulating rolls moves up and the isotherm patterns changes significantly indicating that the convection is the dominating heat transfer mechanism in the cavity. It is observed that the left recirculating vortex becomes dominating in the cavity while the right vortex is squeezed thinner and ultimately is divided into two minor corner vortices. The isotherms are also adjusted according to the changes in the flow field and pushed towards the lower part of the right sidewall indicating the presence of a large temperature gradient there.

The variation of the average Nusselt number, Nu, at the constant flux heated surface with Grashof number, Gr, and corrugation frequencies for discrete heat source length L/W = 0.2 investigated, are shown in Fig.7. In general, it is observed that at any particular corrugation frequency, the average Nusselt number increases significantly with Gr due to enhanced convection especially for Grashof number, Gr $>10^4$. For Gr $\leq 10^4$, the average Nusselt number is almost invariant due to the diffusion dominated heat transfer. The variation of the maximum dimensionless temperature on the heated surface, with Gr for all the cases considered here is presented in Fig.8. We have observed that the maximum temperature decreases with Gr = 10^5 and 10^6 as opposed to the variation of the average Nusselt number with Gr. Also we have observed a constant behavior of

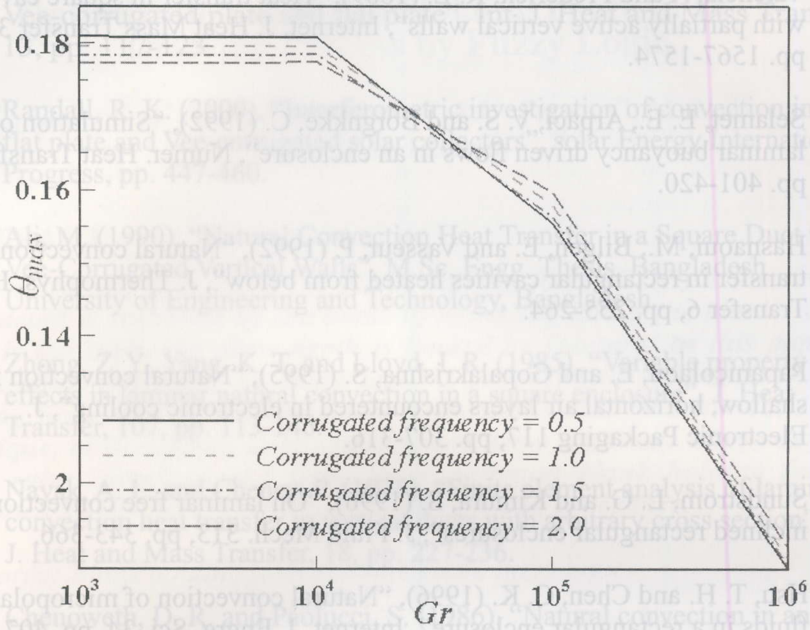

Figure 8: Comparison of the maximum temperature (θ_{max}) for various Grashof numbers and corrugation frequencies (Pr = 0.71)

maximum temperature for different corrugation frequency with Gr = 10^3 and 10^4 which also concludes in Fig. 8. Finally, we see that the maximum heat transfer occurs only in the case of corrugation frequency of 0.5, when Gr increases from 5×10^5. But for any Gr less than 5×10^5. But for any Gr less than 5×10^5, high corrugation frequency dominates the heat transfer rate in a constant manner.

5 CONCLUSIONS

In this paper, natural convective cooling of a localized constant heat flux surface embedded symmetrically at the bottom of a Vee-corrugated enclosure where the top wall is adiabatic and cold sidewalls are isothermal has been investigated and analyzed numerically for various Grashof numbers and corrugation frequencies. The above analysis shows that the Nusselt number increases with the increase of Grashof number. Also the maximum temperature decreases with the increase of corrugation frequency and the maximum heat transfer obtained for corrugated frequency 0.5.

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Optimization of Blank Holding Force in Deep Drawing Process by Fuzzy Logic

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ABSTRACT

In deep drawing process, if the blank holding force (BHF) is too low, the draw depth is restricted by the limits of wrinkling and if the blank holding force is too high, the draw depth is limited by fracture. In this paper an optimization technique by fuzzy logic is proposed to optimize the blank holding force so that the maximum draw depth can be achieved. In this technique, at first blank holding force and draw depth are inter-related by fuzzy rules (patches) and from there optimum blank holding force is computed by standard mathematical curve fitting procedure.

Keywords: Blank holding force, optimization, fuzzy logic.

NOMENCLATURE

F	punch force
P_H	blank holder pressure
c	an empirical factor ranging from 2 to 3
DR	draw ratio (draw depth/punch diameter)
d_0	blank diameter
S_u	ultimate tensile strength
σ_r	radial stress
R_d	radius of the die
β	empirical factor ranging from .02 - .08
K	shear yield stress
μ	friction co-efficient (lets take 0.1)
R_P	radius of the punch
t	thickness of the sheet
r_j	radius of job
r_p	punch corner radius,
r_d	die corner radius,
r_o	is blank radius,
C	clearance

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