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Performance Analysis of Cooperative Relaying in Nakagami-m Fading Channels

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ABSTRACT

This paper is concerned with the analysis of exact Symbol Error Probability (SEP) for cooperative diversity using Amplify-and-Forward (AF) relaying over independent and non-identical Nakagami-m fading channels. The mathematical formulations for Probability Density Function (PDF) and Moment Generating Function (MGF) of a cooperative link have been derived for calculating Symbol Error Probability with well-known MGF based approach taking M-ary Phase Shift Keying (MPSK) signals as input. The numerical results obtained from the proposed mathematical formulations have been compared with different fading conditions. It is observed that the existence of the diversity link in a relay network plays a dominating role in error performance.

Keywords: *Symbol Error Probability, Probability Density Function, Moment Generating Function, Nakagami-m fading.*

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1 INTRODUCTION

Dual hop cooperative relaying recently draws a considerable attention in wireless communications due to its low power requirement, large coverage as well as diversity [1]-[7]. The diversity provides robustness in system performance against variation in fading. In relay communication non-regenerative or Amplify-and-Forward (AF) relaying technique is the most promising one compared to Decode-and-Forward relaying due to its less complexity. In case of AF relaying, relay simply amplifies the received signals and retransmits towards the destination node. In particular, to scale the diversity order with the number of co-operative relays, multiple AF transmissions require orthogonalizing the relay channels.

Nakagami channel has captured the attention of many researchers [8]–[10] for several reasons. First, Nakagami distribution has a relatively simple analytical form, making it attractive in performance analysis. The more important feature, however, is its flexibility in that it can be used to account for both severe and weak fading and includes the classical Rayleigh fading as a special case.

SEP analysis is one of the crucial performance measures for any wireless systems. Previously published theoretical works on SEP analysis in Nakagami-m fading channel deal with the approximate analysis to avoid mathematical complexity. To the best of author’s knowledge, exact analysis of SEP in Nakagami-m fading channels is a major task to the researcher of this field. Outage and Error probability analysis for blind relay has been proposed in [3]-[6] for Rayleigh and Nakagami-m fading channels. Lower bound on error probability is analyzed in [3] and [7] and upper bound in [4] and [11]. Asymptotic analysis of SEP has been performed in [12]-[14] especially for Rayleigh fading case. The authors [15] have found the exact analysis of SEP for differential binary modulation over Nakagami-m fading channels.

In this paper, we derive the exact SEP for Nakagami-m fading channels in case of local channel state information (CSI) assisted AF relaying considering MPSK signaling scheme. To obtain the maximum co-operative diversity, we use orthogonalized relay channels and MRC combiner.

2 SYSTEM MODEL

We consider a dual hop, half duplex, K-cooperative relay network in which source node, relay node and destination node are modeled as single antenna. All the relays are considered in AF mode in Nakagami-m fading channels as shown in Fig.1. We also consider that all the relays have its local (forward and backward channel) CSI only and use equal power in relay nodes irrespective of

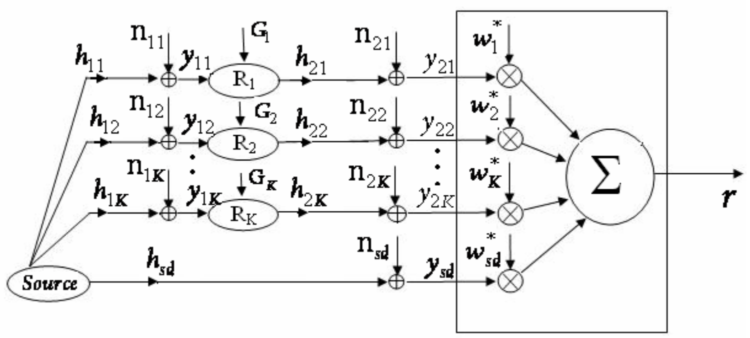

Figure 1: System model

its forward channel and a direct link exists between source and destination nodes. Fading parameters are assumed to be independent and non-identical.

3 SIGNAL MODEL

In the first hop, the source simply broadcast the message and in the second hop, the relays operate in orthogonal channels in time to forward the message to the destination. The receiver has the perfect knowledge of all the relay links, which in turn focused to use MRC technique in the receiver to receive the signal with maximum cooperative diversity.

The received signals at the destination and relay are given in Eq.1 and Eq.2 respectively.

$$y_{sd} = h_{sd}\sqrt{P_s}\,x + n_{sd} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \tag{1}$$

$$y_{1i} = h_{1i}\sqrt{P_s}\,x + n_{1i} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \tag{2}$$

where x is the transmit signal with energy P_s, h_{sd} is complex fading amplitude of the direct link and i is the index of relay node.

The received signal at the destination during the second hop is given by

$$y_{2i} = G_i h_{1i} h_{2i}\sqrt{P_S}\,x + G_i h_{2i} n_{1i} + n_{2i} \quad \dots \quad \dots \quad \dots \quad \dots \tag{3}$$

where h_i is the complex fading amplitude of i^{th} link, 1 and 2 indicate first and second hop respectively. The noise parameters n_{sd}, n_{1i} and n_{2i} are modeled as Zero Mean Circularly Symmetric Complex Gaussian (ZMCSCG) with variances N_{sd}, N_{1i} and N_{2i} respectively.

The received signal in Eq.2 is scaled by gain parameter G_i such that energy from each node is constrained to P_R such that, $P_i \le P_R$ where, P_i is the transmitting power of the i^{th} node and P_R is the maximum power that can be used by a relay.

The gain parameter of i^{th} relay is given by

$$G_i = \sqrt{\frac{P_i}{(\left|h_{1i}\right|^2 P_S + N_{1i})}} \qquad \cdots \quad \cdots \quad \cdots \quad \cdots \tag{4}$$

The output after combining is given as

$$r = w_{sd}^{*} h_{sd} \sqrt{P_S}\, x + \sum_{i=1}^{K} w_i^{*} G_i h_{1i} h_{2i} \sqrt{P_S}\, x + \sum_{i=1}^{K} \left(w_i^{*} G_i h_{2i} n_{1i} + w_i^{*} n_{2i} \right) \qquad \cdots \quad \cdots \tag{5}$$

where $w_{sd}^{*} = \dfrac{h_{sd}^{*}}{N_{sd}}$ and $w_i^{*} = \dfrac{G_i h_{1i}^{*} h_{2i}^{*}}{G_i^2 \left|h_{2i}\right|^2 N_{1i} + N_{2i}}$ are the weights for perfect coherency of direct link and relayed signal of i^{th} relay respectively.

The SNR at the output of the combiner can be described as the sum of two SNR's, one is the SNR of direct link signal and other is the equivalent SNR of the relayed signals, as shown in Eq.6.

$$\gamma = \gamma_{sd} + \gamma_{eq} \qquad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \tag{6}$$

where $\gamma_{eq} = \displaystyle\sum_{i=1}^{K} \frac{\gamma_{1i}\gamma_{2i}}{\gamma_{1i} + \gamma_{2i} + 1}$

4 SYMBOL ERROR PROBABILITY (SEP) ANALYSIS

In order to analyze the SEP of Nakagami-m fading channels, one need to calculate the Probability Density Function, the Cumulative Distribution Function (CDF) and the Moment Generating Function of γ_i.

where γ_i is defined in Eq.7.

$$\gamma_i = \frac{\gamma_{1i}\gamma_{2i}}{\gamma_{1i} + \gamma_{2i} + 1} \qquad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \tag{7}$$

Assuming a random variable, $g_k = \dfrac{g_1 g_2}{g_1 + g_2 + 1}$, where γ_1 and γ_2 are the Nakagami-m distributed random variables with PDF as

$p_\gamma(\gamma) = \dfrac{m^m \gamma^{m-1}}{\bar{\gamma}^{\,m}\Gamma(m)} e^{-\left(\frac{m\gamma}{\bar{\gamma}}\right)}, \gamma \geq 0, m \geq \dfrac{1}{2}$ and $\bar{\gamma}$ is the average SNR.

Then the CDF of γ_k is

$$\begin{aligned}F_{\gamma_k}(\gamma) = 1 - \frac{2m_2^{m_2}(m_1-1)!\; e^{-\left(\frac{m_1\gamma}{\bar{\gamma}_1}+\frac{m_2\gamma}{\bar{\gamma}_2}\right)}}{\bar{\gamma}_2^{\,m_2}\;\Gamma(m_1)\Gamma(m_2)} & \sum_{q=0}^{m_1-1}\sum_{l=0}^{q}\sum_{r=0}^{m_2-1}\left\{\frac{1}{q!}\binom{q}{l}\right. \\ \times\binom{m_2-1}{r}\left(\frac{m_1}{\bar{\gamma}_1}\right)^{\frac{(2q-l+r+1)}{2}} & \left(\frac{m_2}{\bar{\gamma}_2}\right)^{\frac{(l-r-1)}{2}} \gamma^{\frac{(2q+2m_2-l+r+1)}{2}} \\ & \left.\times(\gamma+1)^{\frac{(l+r+1)}{2}} K_{l-r-1}\left(2\sqrt{\frac{m_1m_2\gamma(\gamma+1)}{\bar{\gamma}_1\bar{\gamma}_2}}\right)\right\} \qquad \ldots \quad \ldots \quad (8)\end{aligned}$$

where $K_x(.)$ is the x^{th} – order modified Bessel function of the second kind [6].

The PDF of γ_k can be written as

$$\begin{aligned}p_{\gamma_k}(\gamma) = -\alpha_2\, \{\delta_1 & \sum_{q=0}^{m_1-1}\sum_{l=0}^{q}\sum_{r=0}^{m_2-1}\left\{\frac{1}{2q!}\;(\delta_2\, K_{l-r-1}(\alpha_1)) + \frac{1}{2q!}(\delta_3\, K_{l-r-1}(\alpha_1))\right. \\ & \left. + \frac{1}{2\sqrt{\frac{m_1m_2\gamma(\gamma+1)}{\bar{\gamma}_1\bar{\gamma}_2}}\,q!}(\delta_4\,(-K_{l-r-2}(\alpha_1)) - K_{l-r}(\alpha_1)))\right\}\} \\ & -\alpha_2\,\{\delta_5 \sum_{q=0}^{m_1-1}\sum_{l=0}^{q}\sum_{r=0}^{m_2-1}\{\delta_6\, K_{l-r-1}(\alpha_1)\}\} \qquad \ldots \quad (9)\end{aligned}$$

The description of Eq.9 is illustrated in **Appendix A**.

The MGF can be calculated as follows:

$$\begin{aligned}\Phi_{\gamma_k}(s) &= E\left\{e^{-s\gamma}\right\} \\ &= \int_0^{\infty} e^{-s\gamma} p_{\gamma_k}(\gamma)\; d\gamma \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)\end{aligned}$$

Although it is difficult to solve the Eq.10 analytically, we can solve it numerically.

The exact expression of SEP for the system can be written in well-known MGF based approach for MPSK signaling as Eq.11.

$$P_s(e)=\frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}}\Phi_{\gamma}\left(\frac{Sin^2\frac{\pi}{M}}{Sin^2\theta}\right)d\theta$$

$$=\frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}}\left(1+\frac{\sin^2(\pi/M)\bar{\gamma}_{sd}}{m_{sd}\sin^2\theta}\right)^{-m_{sd}}\Phi_{\gamma_{eq}}\left(\frac{\sin^2(\pi/M)}{\sin^2\theta}\right)d\theta \qquad \ldots \qquad \ldots (11)$$

5 NUMERICAL RESULTS

Numerical results of this paper are based on equal power allocation in all relay nodes. Power of all relay nodes are taken as $P_t/2K$ and $P_s=P_t/2$, where P_t indicates the total power constrained of the system. The noise variances of all hops are taken as unity.

Fig.2, Fig.3 and Fig.4 show the SEP as a function of SNR for different values of fading parameter in Nakagami-m fading channel with two (K=2), five (K=5) and ten (K=10) relays respectively. The subscripts 0, 1 and 2 in the fading parameter m denote the direct link, first hop and second hop fading respectively.

From Fig.2, Fig.3 and Fig.4, one can see that the SEP of a channel is highly affected by the fading and decreases with the decrease in severity of fading (as m goes from 1 to ∞). We also observe that the SEP performance is found to be dependent on the number of relays. As the number of relays increases, the SEP performance of a channel improves due to the improvement in diversity. The change of SEP performance in Rayleigh fading (m = 1) channel with number of relays is more significant. So one can improve the SEP of Rayleigh fading channel or any channel with considerable fading by changing the number of relays in the system. In case of AWGN channel (m = ∞) the effect of the number of relays on the SEP is not so much significant as we see in the Rayleigh fading case. To justify the validity of the present work, in the SEP versus SNR curves as shown in Fig.4, the case when m = 1 (Rayleigh fading) is compared with the result of [16]. It is observed that both the results are similar with each other.

The effects of different fading in a particular relay link (i.e. considering source-relay-destination as a link) have been described in Fig.5, Fig.6 and Fig.7.

In case of two relays as shown in Fig.5, we see that in the direct link (m_0) the SEP performance of the system is highly affected by the fading (change of m_0 for 1 to 5) of the channel. For relayed link, the effect fading on the SEP performance for second hop is higher than that the first hop which is illustrated in Fig.5 by changing m_1 and m_2 from 1, 5 to 5, 1.

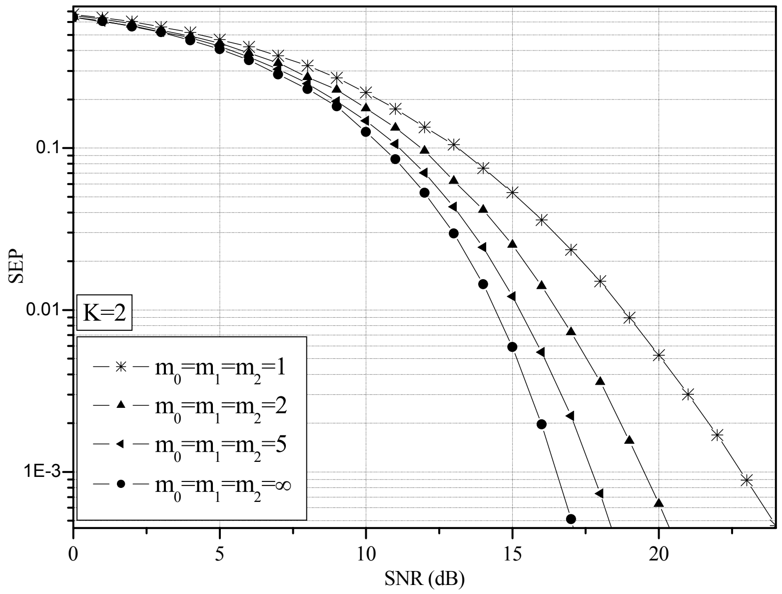

Figure 2: Symbol error probability versus SNR for different Nakagami-m parameter with K=2.

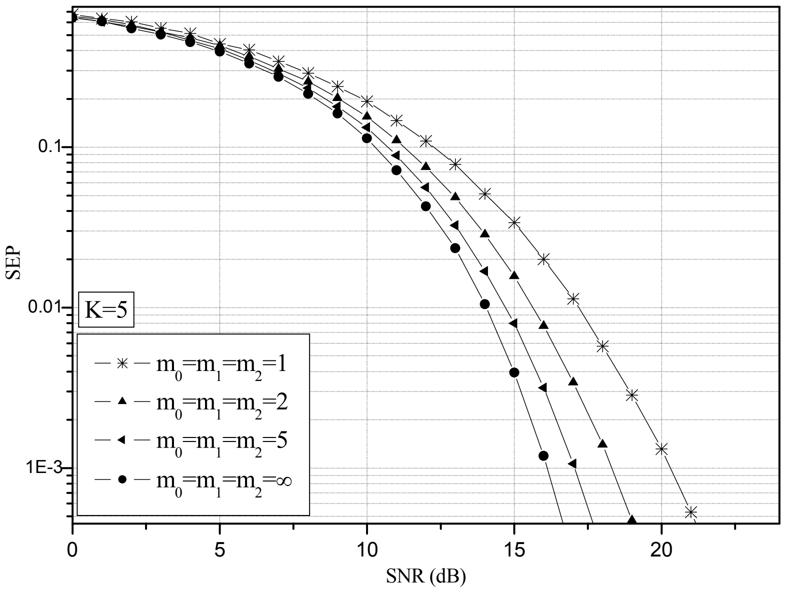

Figure 3: Symbol error probability versus SNR for different Nakagami-m parameter with K=5.

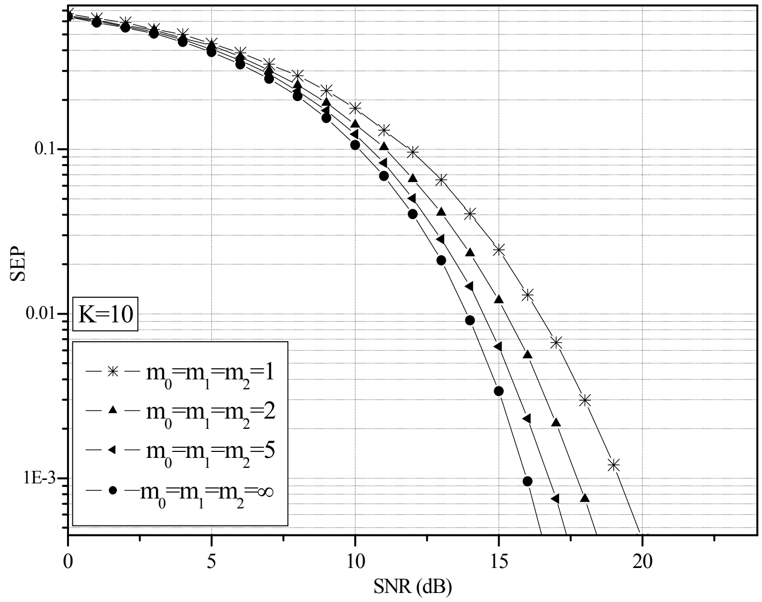

Figure 4: Symbol error probability versus SNR for different Nakagami-m parameter with K=10.

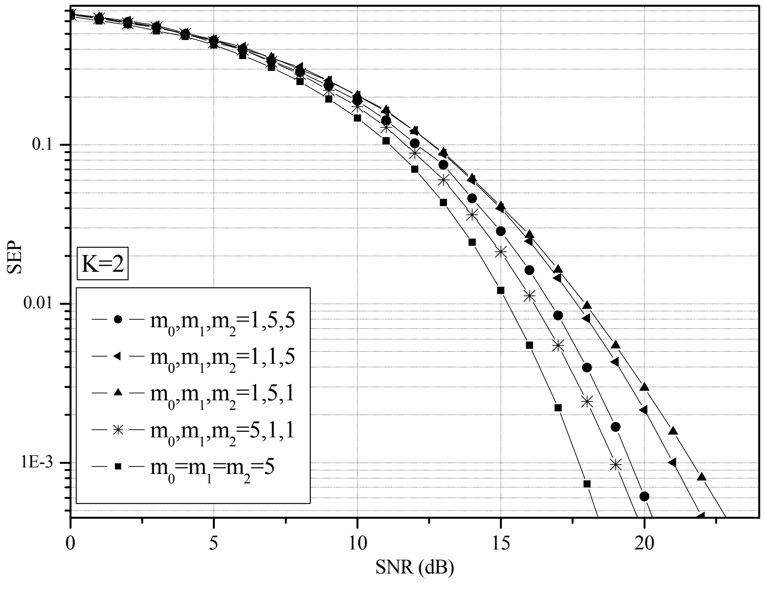

Figure 5: Symbol error probability versus SNR for different Nakagami-m parameter in the particular link with K=2.

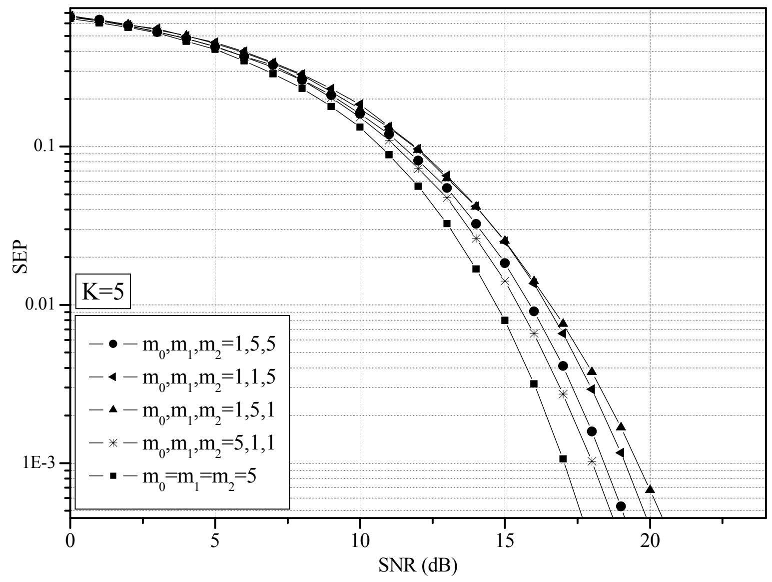

Figure 6: Symbol error probability versus SNR for different Nakagami-m parameter in the particular link with K=5.

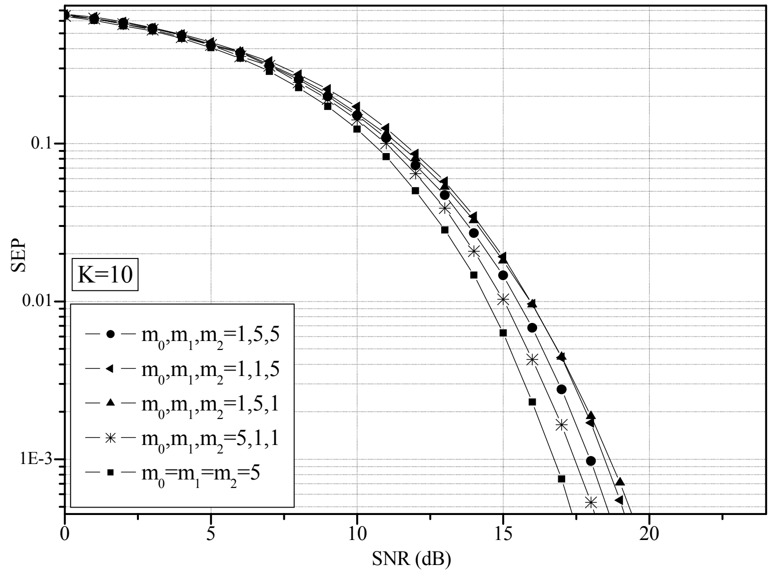

Figure 7: Symbol error probability versus SNR for different Nakagami-m parameter in the particular link with K=10.

The SEP performance in the direct link as well as in the relayed link is affected by the number of relay which can be observed by comparing Fig.5, Fig.6 and Fig.7.

6 CONCLUSIONS

The error performance of a half-duplex AF relay network in Nakagami-m fading has been analyzed. The expression of PDF of received SNR is derived and then using well-known MGF based approach, the SEP of the network is calculated. Simulation results agree that higher values of *m* provide less probability of error. We have also observed that the existence of direct link in different hops plays an important role for reducing error probability. Comparison of SEP in different fading conditions is done simply by varying the Nakagami-m parameter.

APPENDIX-A

The PDF of any distribution can be calculated by differentiating its CDF. So Eq.9 can be obtained from the differentiation of Eq.8 with respect to γ.
Assuming

$$\delta_1 = 2\bar{\gamma}_2^{\,-m_2} e^{-\left(\frac{m_1\gamma}{\bar{\gamma}_1}+\frac{m_2\gamma}{\bar{\gamma}_2}\right)} m_2^{m_2}\left(m_1-1\right)! \qquad \cdots \quad \cdots \quad \cdots \quad \cdots \qquad (9.1)$$

$$\delta_2 = \binom{q}{l}\binom{m_2-1}{r}\left(\frac{m_1}{\bar{\gamma}_1}\right)^{\frac{(2q-l+r+1)}{2}}\left(\frac{m_2}{\bar{\gamma}_2}\right)^{\frac{(l-r-1)}{2}}\left(2q-l-2m_2-r-1\right)$$
$$\times\ \gamma^{\frac{(2q-l+2m_2-r-1)}{2}-1}\left(\gamma+1\right)^{\frac{(l+r+1)}{2}} \qquad \cdots \quad \cdots \, (9.2)$$

$$\delta_3 = \binom{q}{l}\binom{m_2-1}{r}\left(\frac{m_1}{\bar{\gamma}_1}\right)^{\frac{(2q-l+r+1)}{2}}\left(\frac{m_2}{\bar{\gamma}_2}\right)^{\frac{(l-r-1)}{2}}\left(l+r+1\right)$$
$$\times\ \gamma^{\frac{(2q-l+2m_2-r-1)}{2}}\left(\gamma+1\right)^{\frac{(l+r+1)}{2}-1} \qquad \cdots \quad \cdots \quad \cdots \, (9.3)$$

$$\delta_4 = \binom{q}{l}\binom{m_2-1}{r}\left(\frac{m_1}{\overline{\gamma}_1}\right)^{\frac{(2q-l+r+1)}{2}}\left(\frac{m_2}{\overline{\gamma}_2}\right)^{\frac{(l-r-1)}{2}}\gamma^{\frac{(2q-l+2m_2-r-1)}{2}}$$

$$\times\ (\gamma+1)^{\frac{(l+r+1)}{2}}\left(\frac{m_1m_2\gamma}{\overline{\gamma}_1\overline{\gamma}_2}+\frac{m_1m_2(\gamma+1)}{\overline{\gamma}_1\overline{\gamma}_2}\right) \qquad \dots\ \dots \qquad \dots (9.4)$$

$$\delta_5 = 2\overline{\gamma}_2^{\,-m_2}e^{-\left(\frac{m_1\gamma}{\overline{\gamma}_1}+\frac{m_2\gamma}{\overline{\gamma}_2}\right)}m_2^{m_2}(m_1-1)!\left(-\frac{m_1}{\overline{\gamma}_1}-\frac{m_2}{\overline{\gamma}_2}\right) \qquad \dots \qquad \dots \qquad \dots (9.5)$$

$$\delta_6 = \frac{1}{q!}\binom{q}{l}\binom{m_2-1}{r}\left(\frac{m_1}{\overline{\gamma}_1}\right)^{\frac{(2q-l+r+1)}{2}}\left(\frac{m_2}{\overline{\gamma}_2}\right)^{\frac{(l-r-1)}{2}}\gamma^{\frac{(2q+2m_2-l+r+1)}{2}}(\gamma+1)^{\frac{(l+r+1)}{2}} \qquad \dots (9.6)$$

$$\alpha_1 = 2\sqrt{\frac{m_1m_2\gamma(\gamma+1)}{\overline{\gamma}_1\overline{\gamma}_2}} \quad \text{and} \quad \alpha_2 = \frac{1}{\Gamma(m_1)\Gamma(m_2)},$$

we can write the form of PDF of γ_k as shown in Eq.9.

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