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Finite Difference Analysis of Variable Cross-section Fixed-hinged Columns with Non Linearity

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ABSTRACT

A column having variable cross-sections with fixed-hinged end conditions has been modelled assuming that the column material has non-symmetric responses in tension and compression. Recently, a powerful numerical scheme, based on finite difference technique, has been devised by the authors. This method is used to trace the load-deflection curves (equilibrium configuration paths) of the column that has highly non-linear stress-strain curves which are non-symmetric as well in tension and compression. The critical load is determined from the load-deflection curve using theorems of Thompson and Hunt. To utilize the fruitfulness of the devised method, superelastic shape memory alloy (SMA) is used as column materials. Experimentally obtained stress-strain curve of SMA, which is non-symmetric in tension and compression and highly nonlinear, used for determining the buckling response. To make the study more realistic initial shape imperfection has also been included and its effect on column's response has also been discussed in detail.

Keywords: *Buckling, Fixed-hinged, Variable cross-sections, Super elastic Shape memory.*

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1 INTRODUCTION

The present study relies on the self-developed complete computer code which is simple, straightforward and efficient. Such a study will be practically important because tension-compression asymmetry as well as material nonlinearity becomes prominent for large nonlinearly elastic buckling of any short column having varying cross-sections, used in numerous structural applications. It is seen from **Fig.1**, that the tension-compression asymmetry becomes prominent if the strain exceeds 1% for the case of SMA

Buckling analysis of structures inherently involves geometric nonlinearity. It in turn makes it an interesting topic as nonlinear problems usually do not have any closed form solutions. Therefore, if the material nonlinearity (together with non-symmetric responses in tension and compression) is added to the buckling analysis, it then becomes much challenging. Moreover, modern engineering structures are optimally shaped and inherently warrant instability analysis. The present study incorporates all of such points (that is both geometrical and material nonlinearities) and additionally includes varying cross-section of a column. Finally, shape imperfection of the column has also been modelled to make it a very interesting and useful one as far as applications of columns are concerned in adaptive structures.

Euler formula is used to predict critical load of ideal columns having linearly elastic materials; such columns are termed as slender. But, if the critical stresses exceed proportional limit of the column material Euler formula can't be used; we use the term short columns for these specific cases.

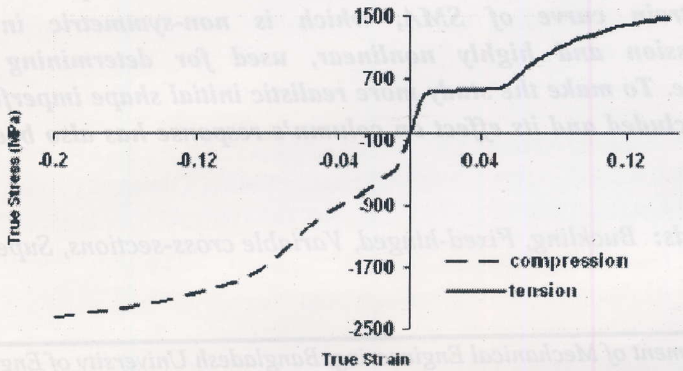

Figure 1: Stress-strain curves for the super elastic SMA specimen (diameter =2 mm) in compression and tension.

Calculation of buckling loads of column by different analytical/numerical schemes is extensively reported in the literature; one of the major reasons of such studies is the discrepancy of results between the experimental buckling loads and their predicted values. Those different numerical methods include finite element, finite difference, as well as strength of materials approach etc. A few of such studies are listed in the reference. For example, Bert and Ko [1] used finite difference technique and calculated buckling loads of columns constructed of bimodular material, which has a different Young's modulus in tension than it has in compression. Gadalla and Abdalla [2] predicted buckling behavior of compression members with variability in material and/or section properties based on eigen solutions. Earlier Li [3] dealt with multi-step non-uniform columns by analytical approach. Virdi [4] used finite difference method for nonlinear analysis of structures. It can be noted that in most of those studies eigen value solutions have been used to predict buckling of columns.

Commercial FEM code ANSYS was used for comprehensive analysis of slender as well as short columns made of stainless steel and shape memory alloy by Rahman et al. [5]-[6] . Of course, constant cross-section has been assumed all along the column span. It was pointed out in those studies that though Euler's slender column formula can be used for ideal cases, inclusions of actual stress-strain relations, which are non-linear, may be necessary if one needs to rigorously study the postbuckling path of a column even for a high slenderness ratio. Moreover, in those of our previous studies, tensile and compressive stress-strain curves were used separately for simulation purpose. Consequently, it was concluded that for numerical predictions of response of short beams/columns made of steel or, shape memory alloy (SMA), simultaneous use of non-linear stress-strain curves in tension and compression becomes essential in some cases (Rahman et al. [5]-[6]);

Of course, commercial FEM code ANSYS has material model like Mooney-Rivlin, that can use both of these tension-compression stress-strain (σ-ε) curves simultaneously during modelling. But substantial modifications of the original σ-ε curves are necessary while evaluating Mooney-Rivlin constants (Rahman and Tani [7]). Since, such modifications are not always desirable; therefore, Rahman, Akanda and Hossain [8] and Hossain [9] used both tensile and compressive stress-strain curves simultaneously. In these studies [8,9], strength of materials approach, termed as Timoshenko's method [10], was used to calculate the buckling load of both ends hinged columns with non-symmetric response in tension and compression. It should be mentioned here that this strength of materials method is suited only for both ends hinged columns having constant cross-section along the column span. A complete study, however, should also include all practically possible boundary conditions of columns.

Observing the above-mentioned facts, the authors developed a new method based on finite difference techniques (Chowdhuri[11], Rahman and Chowdhuri[12]). Based on this method buckling load of column having variable cross-sections is determined for fixed-hinged end condition.

2 MATHEMATICS MODEL

2.1 Governing equation for initially straight column

Taking into account of geometric nonlinearity, the basic fourth order governing equation for the analysis of beam-columns can be given by

$$(EIy^{ii})^{ii} + Py^{ii} = q \quad \cdots \quad \cdots \quad \cdots \quad (1)$$

Where, E, I, y, P and q are modulus of elasticity, area moment of inertia, lateral deflection, axial load, and distributed lateral load, respectively. Superscript 'i' indicates differentiation with respect to independent variable x. Thus, the superscript (^{ii}) means, it is differentiating twice with respect to x.

In order to discretize the governing **Eq. (1)**, using the finite difference expressions for the second order derivative,

$$\frac{d^2 y}{dx^2} = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} \quad \cdots \quad \cdots \quad \cdots \quad (2)$$

The governing equation is obtained as,

$$y_{i+2} - \left(\frac{2a_{i+1} + 2a_i}{a_{i+1}} - A\right) y_{i+1} + \left(\frac{a_{i+1} + 4a_i + a_{i-1}}{a_{i+1}} - 2A\right) y_i$$
$$- \left(\frac{2a_i + 2a_{i-1}}{a_{i+1}} - A\right) y_{i-1} + \frac{a_{i-1}}{a_{i+1}} y_{i-2} = B \quad \cdots \quad (3)$$

Where, h is strip size, a_i is EI at grid i, and $A = \frac{Ph^2}{a_{i+1}}$ and $B = \frac{qh^4}{a_{i+1}}$

For variable cross-sections, the column cross-sections (having constant thickness and variable width) are assumed as

$$I = I_0 \left(1 + \alpha \sin(\pi x / L)\right) \quad \cdots \quad (4)$$

Where, I is moment of inertia, I_0 is the moment of inertia at the end-sections and α is the shape index. Three different shapes of the column are analyzed: a constant width for which $\alpha = 0$, and variable widths for which $\alpha = 0.5$ and 1. The variable cross-section is chosen in this shape because theoretical critical load in linear elastic case for this shape is given by Brush and Almroth [13].

2.2 Governing equation for column with initial shape imperfection

For a column with initial shape imperfection, governing **Eq. (1)** can be rewritten as,

$$(EIy^{ii})^{ii} + P\left(y + y^*\right)^{ii} = q \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

The initial curved shape is arbitrarily assumed as

$$y^* = C_m \sin\left(\frac{\pi x}{L}\right). \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

Where, C_m is specifies amount of initial imperfections.

The discretized governing equation corresponding to **Eq. (5)** for the imperfect column is given below,

$$y_{i+2} - \left(\frac{2a_{i+1} + 2a_i}{a_{i+1}} - A\right)y_{i+1} + \left(\frac{a_{i+1} + 4a_i + a_{i-1}}{a_{i+1}} - 2A\right)y_i$$

$$-\left(\frac{2a_i + 2a_{i-1}}{a_{i+1}} - A\right)y_{i-1} + \frac{a_{i-1}}{a_{i+1}}y_{i-2} = B + Ph^2(y^*_{i+1} - y^*_i + y^*_{i-1}) \quad \ldots (7)$$

2.3 BOUNDARY CONDITIONS

Specified boundary conditions for the columns are given below.

For one end fixed and other end hinged condition,

$$y = 0 = \frac{dy}{dx} \quad \text{at } x = 0 \text{ and } y = 0 = \frac{d^2y}{dx^2} \quad at \quad x = L. \quad \ldots \quad (8)$$

corresponding finite difference equations are,

At $x = 0$,

$$-3y_i + 4y_{i+1} - y_{i+2} = 0 \quad \ldots \quad \ldots \quad (8a)$$

and at $x = L$,

$$2y_i - 5y_{i-1} + 4y_{i-2} - y_{i-3} = 0 \quad \ldots \quad \ldots \quad \ldots \quad (8b)$$

2.4 Incorporation of material nonlinearity in the governing equations

For the linearly elastic columns the elastic modulus E remains constant which makes moment of inertia I as the only variable for the term EI. But In order to calculate nonlinear elastic stiffness ($E'I$) at any cross-section of the column, nonlinear moment-curvature (M-Δ) relation and nonlinear modulus-curvature (E^r-Δ) relation must be known beforehand as far as columns with material nonlinearity are concerned. Therefore, **Eqs. (9)-(11)** are derived based on equilibrium of bending moment and axial force on any cross-section of the column. Details of the derivation can be found in Timoshenko [10] or Hossain [9].

$$\sigma_c = \frac{P}{bh_t} = \frac{1}{\Delta}\int_{\varepsilon_2}^{\varepsilon_1} \sigma \, d\varepsilon \quad \ldots \quad \ldots \quad (9)$$

$$M = \frac{12I}{\rho\Delta^3}\int_{\varepsilon_2}^{\varepsilon_1} \sigma(\varepsilon - \varepsilon_0)\, d\varepsilon \quad \ldots \quad \ldots \quad (10)$$

$$E^r = \frac{12}{\Delta^3}\int_{\varepsilon_2}^{\varepsilon_1} \sigma(\varepsilon - \varepsilon_0)\, d\varepsilon \quad \ldots \quad (11)$$

Where, h_t is the constant thickness of column, σ_c is compressive stress, M is bending moment, ε_1 and ε_2 are the strains at tension and compression side respectively. Δ is the sum of ε_1 and ε_2, ρ is radius of curvature and it is equal to h_t/Δ.

In this study, M-Δ relation and E^r-Δ relation are based entirely on the experimentally obtained stress-strain curve as in **Fig.1**.

3 RESULTS AND DISCUSSION

Since, analysis of these columns requires the nonlinear moment-curvature (M-Δ) and nonlinear modulus curvature (E^r-Δ) relations, two such representative curves are shown in **Fig.2** and **Fig.3**. As seen E^r decreases gradually for increasing values of Δ.

Fig.4 shows the variation of initial values of E^r with P for a particular cross-section. E^r decreases notably only when the axial load exceeds 1700kN, but more significantly when axial load exceeds 2300kN. All the load-deflection curves for the nonlinearly elastic columns having constant cross-sections are obtained with the aid of **Fig.4**. Many such curves are necessary for a single column having variable cross-sections that are handled by the computer code. Interested readers may refer to Chowdhuri [11] for more detail of this point.

The problem of finding E^r becomes more tedious when the cross-section is variable along the column span; a computer program is very useful in this regard.

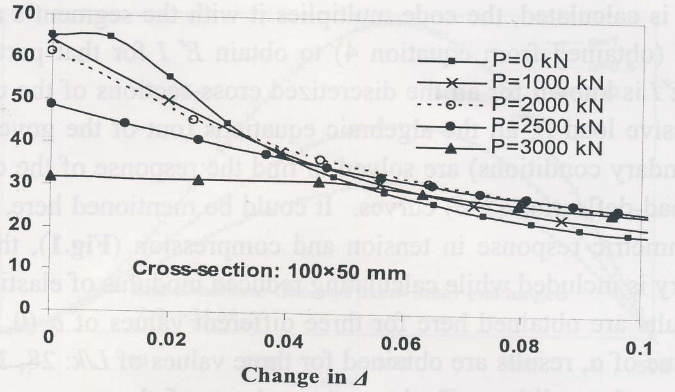

Figure 2: Variation of E' with Δ for SMA column with $\alpha = 0$.

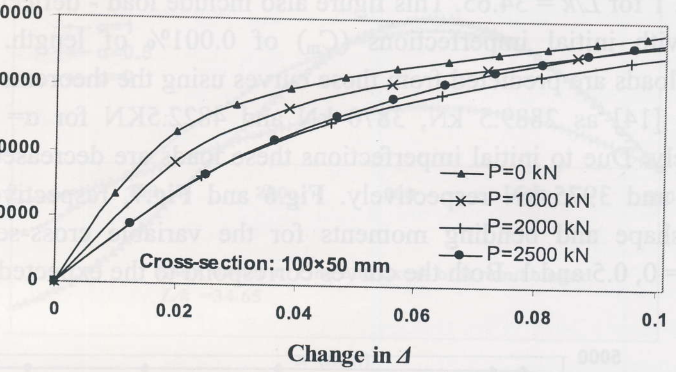

Figure 3: Variation of M with Δ for SMA column with $\alpha = 0$.

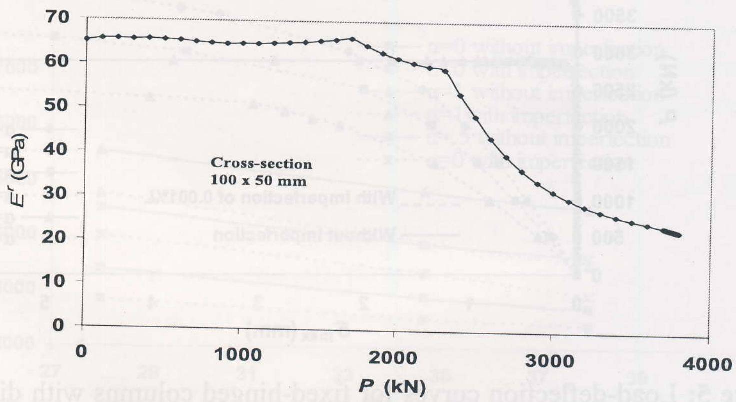

Figure 4: Variation of E' with P for SMA column with $\alpha = 0$.

Once E^r is calculated, the code multiplies it with the segment's area moment of inertia I (obtained from equation 4) to obtain $E^r I$ for that particular segment. When, $E^r I$ is known for all the discretized cross-sections of the columns for any compressive load P, all the algebraic equations (out of the governing equations and boundary conditions) are solved to find the response of the column in terms of the load-deflection (P-δ) curves. It could be mentioned here, since SMA has non symmetric response in tension and compression (**Fig.1**), the effect of non symmetry is included while calculating reduced modulus of elasticity, E^r.

Results are obtained here for three different values of α (0, 0.5 and 1). For each value of α, results are obtained for three values of L/k: 28, 34.65 and 38 for different end conditions. To keep the volume of the paper a minimum, only representative results are shown and discussed below:

Figs.5 show the load - deflection patterns for fixed-hinged columns with α= 0, 0.5 and 1 for L/k = 34.65. This figure also include load - deflection curves for column with initial imperfections (C_m) of 0.001% of length. From **Fig.5**, Buckling loads are predicted from these curves using the theorems of Thompson and Hunt [14] as 2889.5 kN, 3870 kN and 4822.5KN for α= 0, 0.5 and 1 respectively. Due to initial imperfections these loads are decreased to 2530 kN, 3350 kN and 3975 kN respectively. **Fig.6** and **Fig.7**, respectively, show the buckled shape and bending moments for the variable cross-section column having α = 0, 0.5 and 1. Both the curves correspond to the expected shapes.

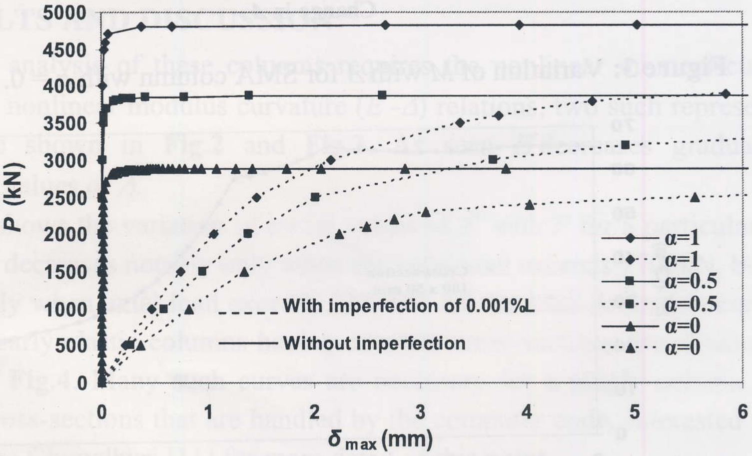

Figure 5: Load-deflection curves for fixed-hinged columns with different α and L/k = 34.65

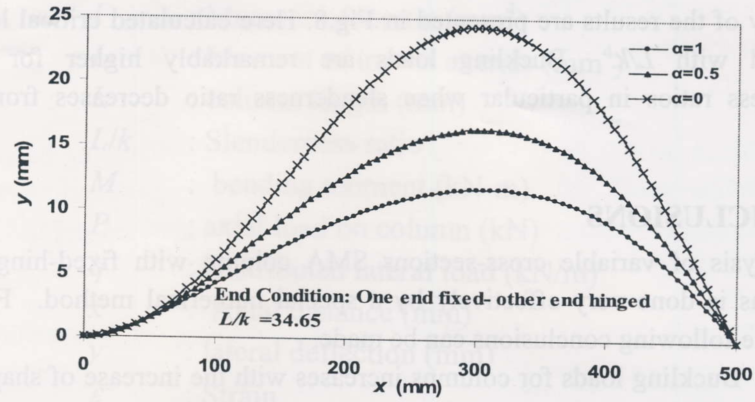

Figure 6: Deflected shape of fixed-hinged columns with different α and $L/k = 34.65$.

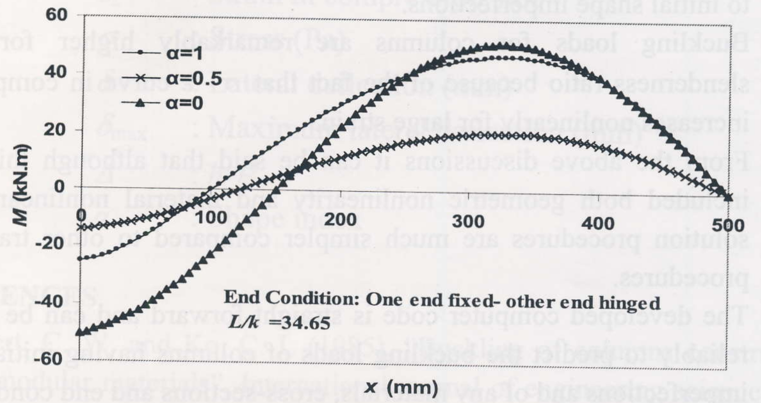

Figure 7: Moment curve of fixed-hinged columns with different α and $L/k = 34.65$

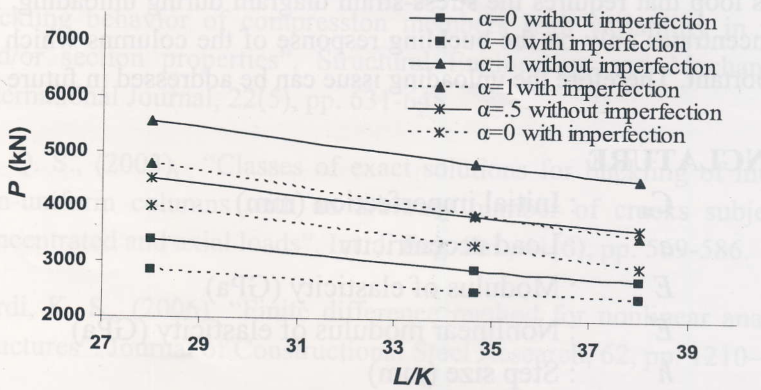

Figure 8: Variation of critical load with L/k for fixed-hinged SMA column.

Summary of the results are presented in **Fig.8**. Here calculated critical loads are presented with *L/k*. Buckling loads are remarkably higher for smaller slenderness ratios in particular when slenderness ratio decreases from 34.65 to 28.

4 CONCLUSIONS

Analysis of variable cross-sections SMA column with fixed-hinged end-conditions is done very effectively by a special numerical method. From the results the following conclusions can be made,

(i) Buckling loads for columns increases with the increase of shape index (α) because moment of inertia is increased with α.

(ii) As expected buckling loads for columns are decreased remarkably due to initial shape imperfections.

(iii) Buckling loads for columns are remarkably higher for lower slenderness ratio because of the fact that σ - ε curve in compression increases nonlinearly for large strain.

(iv) From the above discussions it can be said that although this study included both geometric nonlinearity and material nonlinearity, the solution procedures are much simpler compared to other traditional procedures.

(v) The developed computer code is straight-forward and can be applied reliably to predict the buckling loads of columns having initial shape imperfections and of any materials, cross-sections and end conditions.

Usually superelastic SMA can recover large strain through a typical hysteresis loop that requires the stress-strain diagram during unloading. But this study concentrates only on the buckling response of the columns which itself is quite important. Therefore the unloading issue can be addressed in future studies.

NOMENCLATURE

C_m : Initial imperfection (mm)
e : Load eccentricity
E : Modulus of elasticity (GPa)
E^r : Nonlinear modulus of elasticity (GPa)
h : Step size (mm)
h_t : Thickness of column (mm)

I : Moment of inertia (mm^4)
I_0 : Moment of inertia at ends (mm^4)
L : column length (mm)
L/k : Slenderness ratio
M : bending moment (kN-m)
P : axial load on column (kN)
q : Distributed lateral load (kN/m)
x : Axial distance (mm)
y : lateral deflection (mm)
ε : Strain
ε_1 : Strain in tension side
ε_2 : Strain in compression side
σ : Stress (Pa)
δ : Lateral deflection (mm)
δ_{max} : Maximum lateral deflection (mm)
Δ : h/ρ
α : Shape index

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DYNAMIC EQUIVALENCE OF A LARGE POWER SYSTEM USING POWER SYSTEM SIMULATOR FOR ENGINEERS (PSS/E)

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ABSTRACT

In this paper the step-by-step procedure of obtaining the network equivalent of a large power system using Power System Simulators for Engineers (PSS/E) is presented. Coherency among the generators of the study system is identified using the non-linear time domain simulation obtained by PSS/E. Generators with the most identical swing are considered to be coherent. Dynamic aggregation of the coherent group of generators is performed based on the Zhukov's method. The accuracy of the procedure is demonstrated by comparing the steady state and dynamic results of the original and the equivalent system. The comparisons clearly indicate excellent level of accuracy achieved from this work. The step-by-step procedure of building dynamic equivalent presented in this paper will be extremely helpful for the researchers to understand and work with the commercial PSS/E software.

Keywords: Dynamic equivalent, Coherency identification, Dynamic Aggregation.

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