JOURNAL OF ENGINEERING AND TECHNOLOGY, Vol. 8 No. 2 December 2010 ISSN 1684-4114
http://www.iutoic-dhaka.edu/jet

Spectral and Statistical Analysis of Human Sleep EEG Signals

Md. Shahedul Amin*, Md. Riyasat Azim**, A.S.M. Shihavuddin**, Md. Ashraful Hoque**

Received 21 December 2010; Accepted after revision 31 December 2010

ABSTRACT

Electroencephalogram (EEG) is a complex signal resulting from postsynaptic potentials of cortical pyramidal cells and an important brain state indicator with specific state dependent features. Modern brain research is intimately linked to the feasibility to record the EEG and to its quantitative analysis. EEG spectral analysis is an important method to investigate the hidden properties and hence the brain activities. Spectral analysis of sleep EEG signal provides acute insight into the features of different stages of sleep which can be utilized to differentiate between normal and pathological conditions. This paper describes the process of extracting features of human sleep EEG signals through the use of tools used for spectral analysis of conventional signals. This paper also discusses statistical analysis of human sleep EEG signals in order to detect any hidden patterns lying within the human sleep EEG signal of same kind and nature. To justify the results obtained from applying the spectral and statistical analysis tools K-Means algorithm and Receiver Operating Characteristics have been used. It analyzes the accuracy of the system and defines the relationship between the accuracy, reliability and the amount of test data. This paper also discusses the clinical correlation associated with sleep EEG signals in brief.

Keywords: *EEG, Receiver Operating Characteristics, K-Means Algorithm, Power Spectral Density.*

* *Dept. of Electrical and Computer Engineering, University of Alberta, Canada.*
** *Electrical and Electronic Engineering (EEE) Department, Islamic University of Technology (IUT), Gazipur-1704, Bangladesh. E-mail: riyasat39@gmail.com*

1 INTRODUCTION

EEG is the recording of electrical activity along the scalp produced by the firing of neurons within the brain [1]. The EEG signal has been used as a diagnostic tool for a long time to analyze the activity of the brain and is playing a very important role in diagnosis of diseases and health disorders. Various models have already been suggested to describe the way that the brain produces the EEG signal [2]. The physiological investigation of sleep implies the acquisition and the study of several types of signals. The polysomnographic recordings allow analyzing at the same time the organization of sleep in stages and cycles and in a finer way, the microstructures of the registered signals. The brain activities are characterized by their frequency, amplitude, morphology, stability, topology and ability to react. They are classified according to their wave band. These constituents constitute the microstructure of the sleep and the stage of sleep is largely identified from the microstructure. The human sleep EEG has the characteristic waveform pattern according to each sleep stage; Stage W, 1, 2, 3, 4 and REM. And many studies have been carried out on the automatic human sleep stage determination systems based on the standard rule proposed by the Association for Psycho Physiological Study of Sleep (Rechtshaffen and Kales sleep scoring rules)[3][4]. However, the system is insufficient for extracting more detail information about sleep stages. Therefore, analysis of the EEG signal in the time as well as in the frequency domain by using the detailed spectral analysis was given more interest.

This paper deals with the analysis of sleep EEG signals of human beings. The sleep EEG signals can be used to identify disorders and anomalies of a patient. The healthy sleep EEG patterns are matched with the patterns under scrutiny and can be analyzed for abnormalities. Various kinds of neurological diseases can be identified with the help of sleep EEG signals. Sleep disorders, sleep apnea, mental distress, epilepsy, tumors, cerebrovascular and other brain lesions etc. are a few prominent names. The sleep EEG analysis can have prominent applications for sleep apnea patients, monitoring awareness during the anesthesia and even for paralyzed patients. So the techniques and results described in this paper can have a very important part in modern medical facilities to improve the diagnosis tool.

Sleep EEG signals from PhysioNet database [5] are used for the analysis. A total of 129 EEG signals out of which 68 signals were sleep EEG signals and the rest 61 signals were EEG signals from wake stage and they were closely observed and analyzed in this paper. A very useful tool in signal analysis –

spectral analysis was chosen first for the analysis. Then variation in magnitude of EEG signals in various frequency bands of sleeping persons was utilized for finding out a concluding index. Therefore the aim of the paper is to establish a relationship between the various stages of human sleep and the EEG waves of different frequency bands. Different statistical parameters were also used for finding out a factor for concluding about the results. The accuracy of the obtained results was also justified using k-means clustering algorithm and Receiver Operating Characteristic (ROC) curve by varying the number of learning data and test data.

A combination of the above mentioned methods provides acute insight into the properties of different stages of sleep which can be utilized to identify disorders. Matlab7.5.0 along with "EEGLAB" toolbox and Sigview were used for the necessary analysis.

2 MATERIALS AND METHODS

2.1 Data Collection

A dataset containing 129 full overnight polysomnograms from adult subjects is used for the analysis which was available from the PhysioNet Bank database [5]. The dataset contained 68 EEG signals from various stages of sleep and 61 EEG signals from wake stage. Polysomnograms from Subjects with no known cardiac disease, autonomic dysfunction, and not on medication known to interfere with heart rate were included in the data set.

Subjects were selected randomly (Age: 50 ± 10 years; Body Mass Index (BMI): 31.6 ± 4.0 kg/m², range 25.1-42.5 kg/m²; Apnea Hypopnea Index (AHI): 24.1 ± 20.3, range 1.7-90.9). Polysomnograms were obtained using the Jaeger-Toennies system (Erich Jaeger GmbH, Germany). Signals recorded were: EEG (C3-A2), EEG (C4-A1), left EOG, right EOG, submental EMG, ECG (modified lead V2), Oro-nasal airflow (thermistor), ribcage movements, abdomen movements (uncalibrated strain gauges), oxygen saturation (finger pulse oximeter), snoring (tracheal microphone) and body position. The polysomnograms were scored by an experienced sleep specialist.

2.2 EEG Pre-processing

EEG signals obtained from PhysioNet Bank database were sampled at 128 Hz. The DC offset in EEG signals is removed by using a Finite Impulse Response (FIR) High pass filter (HPF) of order 200. The DC offset free EEG signals were further filtered using FIR Band Pass Filter (BPF) with cut-off

(corner) frequencies of 0.15 - 50 Hz. The output of the filter was then reversed and passed through the filter again in order to realize a zero-phase digital filter. The EEG signals were divided into 10 second segments (**Fig.1**) within and around the time of different sleep stages scored by the sleep specialist and annotated in the database of PhysioNet Bank for further analysis.

2.3 Data Analysis Methodology

2.3.1 Spectral Analysis

EEG spectrum is divided into five main rhythms. Delta waves, slowest EEG rhythms and generally have the highest amplitude observed in EEG waveforms (about 300 μV) with all the frequencies in the range of 0.25 to 4 Hz. Theta waves are typically of smaller amplitude and higher frequency than delta waves. Their frequency range is normally between 4 and 8 HZ. Alpha waves, which occur during relaxed states, are regular rhythms of 8 to 12 Hz with lower amplitude than delta and theta waves but higher amplitudes than sigma and beta waves. Sigma waves are rhythms with frequencies in the range of 12 to 16 Hz and finally Beta waves, defined as low voltage (around 5 μV) and high frequency waves (14 to 40 Hz, sometimes as high as 50 Hz).The different sleep stages are characterized by the presence of certain EEG rhythms while by the absence of others [6, 7]. The power spectral density (**Fig.2**) estimate was calculated using the Welch Method of averaged periodograms for in the entire frequency range of 1-50 HZ and also for the particular EEG rhythms (delta, theta, alpha, sigma & beta with the defined frequency ranges of 0.25-4 Hz, 4-8 Hz, 8-12 Hz, 12-16 Hz, and 16-40 Hz respectively). Average power of delta, theta, alpha, beta and sigma frequency bands was calculated. In order to calculate the average power for individual frequency bands, elliptical bandpass filters with appropriate passband and cutoff frequencies were used to separate the particular bands from the EEG data and then average power within that particular frequency band was calculated. The values of the power spectral estimate obtained for each frequency band was then normalized by the power spectral estimate of the signal in the entire frequency range from 1-50 Hz.

2.3.2 Statistical Analysis

The mean, indicated by μ, is the statistician's jargon for the average value of a signal. Mathematically the mean can be expressed as

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

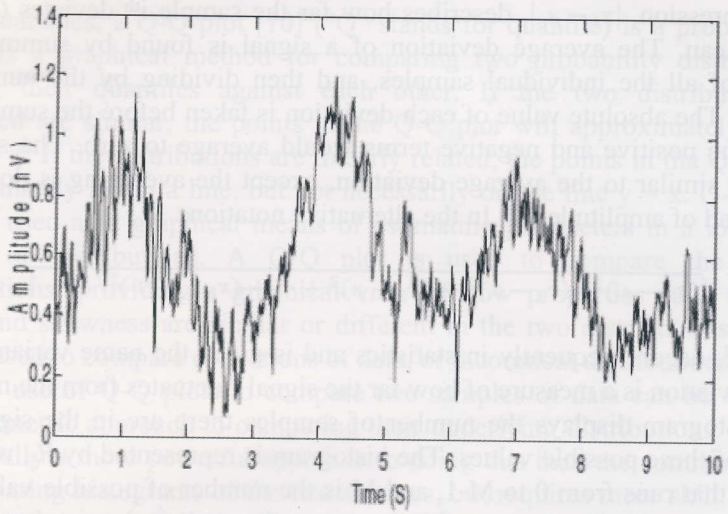

Figure 1: EEG signal segment containing 10 seconds of data.

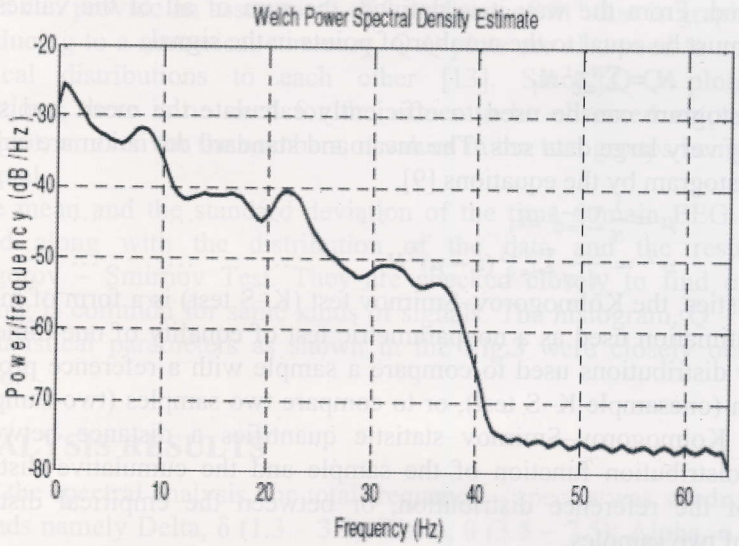

Figure 2: Power spectral density estimate of EEG signal segment.

A more generalized parameter called the standard deviation, denoted by σ is given by the equation,

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=0}^{N-1}(x_i - \mu)^2} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

The expression, $|x_i - \mu|$, describes how far the sample i^{th} deviates (differs) from the mean. The average deviation of a signal is found by summing the deviations of all the individual samples, and then dividing by the number of samples, N. The absolute value of each deviation is taken before the summation; otherwise the positive and negative terms would average to zero. The standard deviation is similar to the average deviation, except the averaging is done with power instead of amplitude [8]. In the alternative notation,

$$\sigma = \sqrt{(x_0 - \mu)^2 + (x_1 - \mu)^2 + (x_2 - \mu)^2 + \cdots + (x_{N-1} - \mu)^2 / N - 1} \quad \ldots \quad (3)$$

The term σ^2, occurs frequently in statistics and is given the name variance. The standard deviation is a measure of how far the signal fluctuates from the mean.

The histogram displays the number of samples there are in the signal that have each of these possible values. The histogram is represented by H_i, where *'i'* is an index that runs from 0 to M-1, and M is the number of possible values that each sample can take on. Just as with the mean, the statistical noise (roughness) of the histogram is inversely proportional to the square root of the number of samples used. From the way it is defined, the sum of all of the values in the histogram must be equal to the number of points in the signal,

$$N = \sum_{i=0}^{N-1} H_i \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

The histogram can be used to efficiently calculate the mean and standard deviation of very large data sets. The mean and standard deviation are calculated from the histogram by the equations [9],

$$\mu = \frac{1}{N} \sum_{i=0}^{N-1} i H_i \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

$$\sigma^2 = \frac{1}{1-N} \sum_{i=0}^{N-1} (i - \mu)^2 H_i \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

In statistics, the Kolmogorov–Smirnov test (K–S test) is a form of minimum distance estimation used as a nonparametric test of equality of one-dimensional probability distributions used to compare a sample with a reference probability distribution (onesample K–S test), or to compare two samples (two-sample K–S test). The Kolmogorov–Smirnov statistic quantifies a distance between the empirical distribution function of the sample and the cumulative distribution function of the reference distribution, or between the empirical distribution functions of two samples.

Under null hypothesis that the sample comes from the hypothesized distribution$F(x)$, $\sqrt{n}\, D_n \rightarrow \sup_t |B(F(t))|$ in distribution, where B(t) is the Brownian Bridge [9]. If F is continuous then under the null hypothesis $\sqrt{nD_n}$ converges to the Kolmogorov distribution, which does not depend on F. This result is also known as the Kolmogorov Theorem. The goodness-of-fit test or the Kolmogorov Smirnov test is constructed by using the critical values of the Kolmogorov distribution. The null hypothesis is rejected at level α if, $\sqrt{nD_n} > K_\alpha$, where K_α is found from,

$$\Pr(K \leq K_\alpha) = 1 - \alpha \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

In statistics, a Q-Q plot [10] ("Q" stands for quantile) is a probability plot, which is a graphical method for comparing two probability distributions by plotting their quantiles against each other. If the two distributions being compared are similar, the points in the Q-Q plot will approximately lie on the line y = x. If the distributions are linearly related, the points in the Q-Q plot will approximately lie on a line, but not necessarily on the line y = x. Q-Q plots can also be used as a graphical means of estimating parameters in a location-scale family of distributions. A Q-Q plot is used to compare the shapes of distributions, providing a graphical view of how properties such as location, scale, and skewness are similar or different in the two distributions. Q-Q plots can be used to compare collections of data, or theoretical distributions.

The use of Q-Q plots to compare two samples of data can be viewed as a nonparametric approach to comparing their underlying distributions. A Q-Q plot is generally a more powerful approach to doing this than the common technique of comparing histograms of the two samples, but requires more skill to interpret. Q-Q plots are commonly used to compare a data set to a theoretical model [11] – [12].

This can provide an assessment of "goodness of fit" that is graphical, rather than reducing to a numerical summary. Q-Q plots are also used to compare two theoretical distributions to each other [13]. Since Q-Q plots compare distributions, there is no need for the values to be observed as pairs, as in a scatter plot, or even for the numbers of values in the two groups being compared to be equal.

The mean and the standard deviation of the time domain EEG signals are recorded along with the distribution of the data and the results of the Kolmogorov – Smirnov Test. They are checked closely to find out whether something is common for same kinds of signals. The histogram, Q – Q plot and other statistical parameters as shown in the **Fig.3** were closely observed and analyzed.

3 ANALYSIS RESULTS

For the spectral analysis, the total frequency – spectra was subdivided in the five bands namely Delta, δ (1.3 – 3.5); Theta, θ (3.5 – 7.5); Alpha, α (7.5 – 12); Sigma, σ (12 – 14) and Beta, β (14 – 35). All the bands of all the signals are analyzed in detain and maximum value of the band along with the position of maximum value, minimum value of the band along with the position of minimum value and mean of all the bands are closely observed as well as recorded. All the parameters as mentioned are given for a subject in **Table 1**.

It is observed in most of the cases that the ratio of the mean, maximum value and minimum value of the Delta and Theta band as well as the ratio of the Alpha and Sigma band is greater than 1 in the sleep EEG signals whereas these values are less than 1 for wake EEG signals. Along with this, it was also observed that

there is any pattern of discontinuity in the frequency domain of the corresponding EEG signals as depicted in **Table – 1**.

Just like spectral analysis, in this part also, the PSD was observed band by band for every signal. Here only the maximum and minimum values of all the bands are analyzed and compared with complimentary bands. In this case the values are in dB since PSD is calculated in logarithmic scale. Now the Theta band is subtracted from Delta band and Sigma band is subtracted from Alpha band for both the maximum and minimum values for all the signals. It is now observed that, this subtraction result is positive for sleep EEG signals and negative for wake EEG signals for almost all the cases, as expected from the spectral analysis. So the argument stated in the results of the spectral analysis is well established from the PSD analysis **(Table – 2)**.

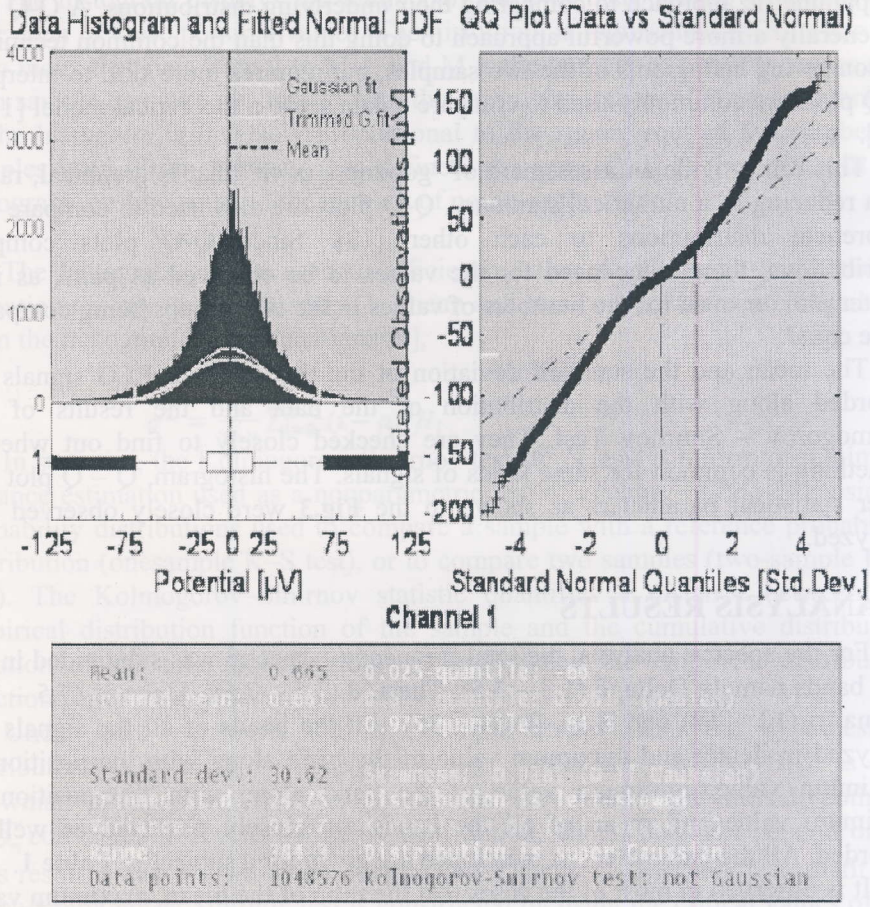

Figure 3: Data Histogram, Q-Q Plot and different statistical parameters of subject EDF (Fpz-Cz).

Table-1: Different Spectral Component of a certain subject

			FREQUENCY BAND (Hz)				
PATIENT NAME	ID	PARAMETER	δ (1.3 - 3.5)	θ (3.5 - 7.5)	α (7.5 - 12)	σ (12 - 14)	β (14 - 35)
AB SALAM	SALAM 1	MAX	8.38E-06	5.81E-06	4.97E-06	4.25E-06	7.03E-06
		MAX POS	3.02	3.90	11.04	13.38	22.26
		MIN	1.07E-06	4.49E-07	6.44E-07	4.48E-07	2.47E+07
		MIN POS	3.32	6.64	8.69	12.99	24.22
		MEAN OF FFT	4.23E-06	2.67E-06	2.88E-06	1.96E-06	2.91E-06
		SPECIALITY			Discontinuity at 9.5, 11.3 & 11.6		Highly discontinuous

Table-2: Different PSD Parameter of a certain subject

			FREQUENCY BAND (Hz)					
PATIENT NAME	ID	PARAMETER	δ (1.3 - 3.5)	θ (3.5 - 7.5)	α (7.5 - 12)	σ (12 - 14)	β (14 - 35)	Distribution of Data
AB SALAM	SALAM 1	MEAN (μV)		125				Data distribution is right skewed and super Gaussian, Kolmogorov Smirnov Test: not Gaussian
		STD.		59.96				
		MAX (dB)	42	24	11	9	6	
		MIN (dB)	24	0	1	-2	-15	

The mean and the standard deviation of the time domain EEG signals are recorded along with the distribution of the data and the results of the Kolmogorov – Smirnov Test. They are checked closely to find out whether something is common for same kinds of signals. The histogram, Q – Q plot and other statistical parameters as shown in the **Fig.3** were closely observed and analyzed. Though same was done for all the subjects but from statistical analysis nothing special was found and the result was random for similar type of subjects.

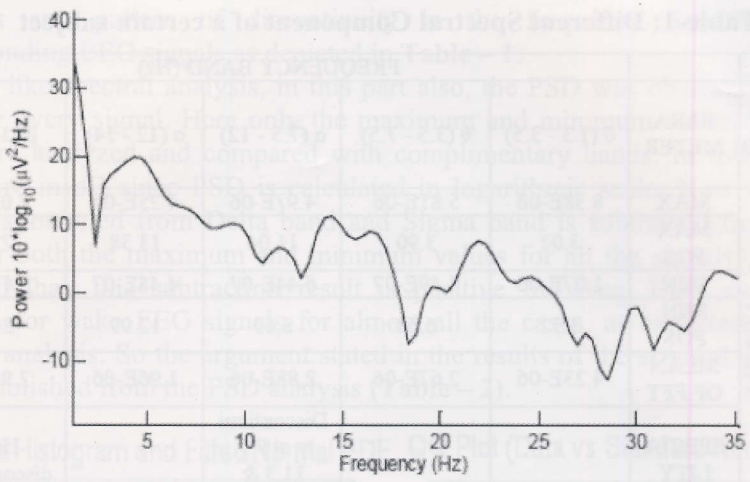

Figure 4: PSD of a certain subject.

4 JUSTIFICATION OF THE RESULTS

4.1 K-Means Algorithm

In statistics and machine learning, *k*-means clustering is a method of cluster analysis which aims to partition n observations into k clusters in which each observation belongs to the cluster with the nearest mean. It is similar to the expectation maximization algorithm for mixtures of Gaussians in that they both attempt to find the centers of natural clusters in the data as well as in the iterative refinement approach employed by both algorithms [14] – [18]. Given a set of observations $x_1, x_2, ..., x_n$ where each observation is a *d*-dimensional real vector, *k*-means clustering aims to partition the n observations into k sets (k < n) $S = S_1, S_2, ..., S_k$ so as to minimize the within-cluster sum of squares (WCSS):

$$\sum_{i=1}^{k} \sum_{x_j \in S_i} \|x_j - \mu_i\|^2 \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

The most common algorithm uses an iterative refinement technique. Due to its ubiquity it is often called the *k*-means algorithm; it is also referred to as Lloyd's algorithm, particularly in the computer science community [19] – [20].

4.2 Receiver Operating Characteristics (ROC)

In signal detection theory, a receiver operating characteristic (ROC), or simply ROC curve, is a graphical plot of the sensitivity, or true positives, vs. (1 – specificity), or false positives, for a binary classifier system as its discrimination

threshold is varied [21] – [23]. The ROC can also be represented equivalently by plotting the fraction of true positives (TPR = true positive rate) vs. the fraction of false positives (FPR = false positive rate). It is known as a Relative Operating Characteristic curve, because it is a comparison of two operating characteristics (TPR & FPR) as the criterion changes [24]. ROC analysis provides tools to select possibly optimal models and to discard suboptimal ones independently from (and prior to specifying) the cost context or the class distribution. ROC analysis is related in a direct and natural way to cost/benefit analysis of diagnostic decision making [25] – [28]. A classification model (classifier or diagnosis) is a mapping of instances into a certain class/group. The classifier or diagnosis result can be in a real value (continuous output) in which the classifier boundary between classes must be determined by a threshold value, for instance to determine whether a person has hypertension based on blood pressure measure, or it can be in a discrete class label indicating one of the classes. Let us consider a two-class prediction problem (binary classification), in which the outcomes are labeled either as positive (p) or negative (n) class. There are four possible outcomes from a binary classifier. If the outcome from a prediction is p and the actual value is also p, then it is called a true positive (TP); however if the actual value is n then it is said to be a false positive (FP). Conversely, a true negative has occurred when both the prediction outcome and the actual value are n, and false negative is when the prediction outcome is n while the actual value is p. Let us define an experiment from P positive instances and N negative instances. The four outcomes can be formulated in a 2×2 contingency table or confusion matrix, as shown in **Fig.5**.

		actual value		
		p	*n*	total
prediction outcome	*p'*	True Positive	False Positive	P'
	n'	False Negative	True Negative	N'
	total	P	N	

Figure 5: Contingency Table of ROC.

The contingency table can derive several evaluation "metrics". To draw an ROC curve, only the true positive rate (TPR) and false positive rate (FPR) are needed. TPR determines a classifier or a diagnostic test performance on classifying positive instances correctly among all positive samples available during the test. FPR, on the other hand, defines how many incorrect positive results occur among all negative samples available during the test. An ROC space is defined by FPR and TPR as x and y axes respectively, which depicts relative trade-offs between true positive (benefits) and false positive (costs). Since TPR is equivalent with sensitivity and FPR is equal to 1 - specificity, the ROC graph is sometimes called the sensitivity vs (1 - specificity) plot. Each prediction result or one instance of a confusion matrix represents one point in the ROC space. The best possible prediction method would yield a point in the upper left corner or coordinate (0,1) of the ROC space, representing 100% sensitivity (no false negatives) and 100% specificity (no false positives). The (0,1) point is also called a perfect classification [29] – [32]. A completely random guess would give a point along a diagonal line (the so-called line of no-discrimination) from the left bottom to the top right corners. The diagonal divides the ROC space. Points above the diagonal represent good classification results, points below the line poor results. The confidence level tells how much sure one can be regarding the obtained results. It is expressed as a percentage and represents how often the true percentage of the population who would pick an answer lies within the confidence interval. The 95% confidence level means one can be 95% certain about the obtained results.

5 DISCUSSIONS

The method is applied to a total of 129 EEG signals. Among them 68 signals are sleep EEG signals and the remaining 61 signals are awake EEG signals. If all the signals are considered as the training signals, then it is found from the ROC curve (as shown in **Fig.6**) that, the system provides 69% accurate results. Now if it is considered that, out of 68 sleep EEG signals 40 are training signals and the remaining 28 are test signals and for awake stage, 40 are training signals and the rest 21 are test signals. Then the system provides 21.3% accurate results. However the result is unacceptable since the false positive area is larger as shown in **Fig.7**.

If it is considered that, out of 68 sleep EEG signals, 50 are training signals and the remaining 18 are test signals and also for awake stages, 50 are training and rest 11 are test signals; then the system provides 59.1% accurate results which is quite acceptable. This is because the true positive area is larger as

shown in **Fig.8**. Again if it is considered that, out of 68 sleep EEG signals 55 are training signals and the remaining 13 are test signals and also for awake stages, 55 are training and the rest 6 are test signals. Then the system provides 59% accurate result which is acceptable. This is because the true positive area is larger as shown in **Fig.9**.

In a nutshell, it can be concluded that, when the number of training signals is higher, then the accuracy of the system increases and when the number of training data is less, then the accuracy of the system decreases as depicted in **Fig.10**. So it is always better for one to find the accuracy of the result obtained considering a large number of training data.

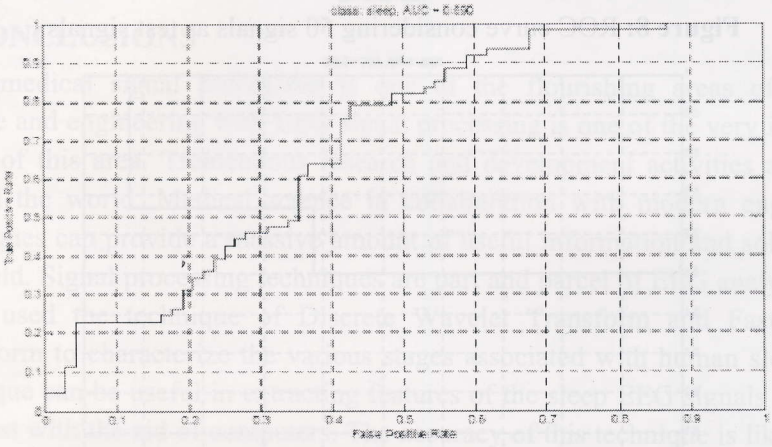

Figure 6: ROC curve considering 129 signals as test signals.

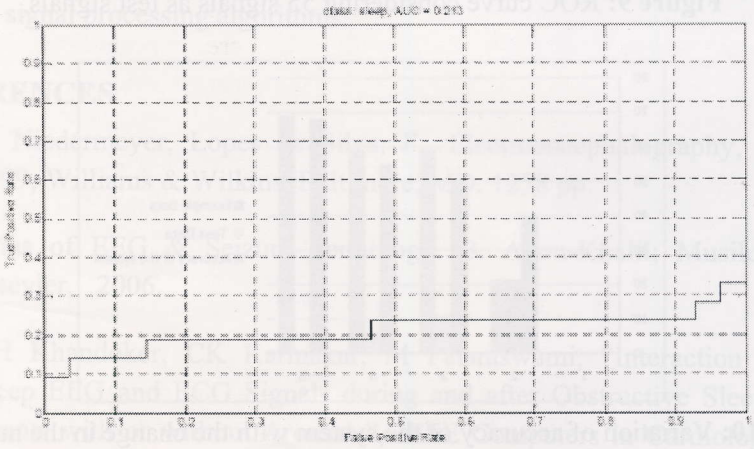

Figure 7: ROC curve considering 40 signals as test signals.

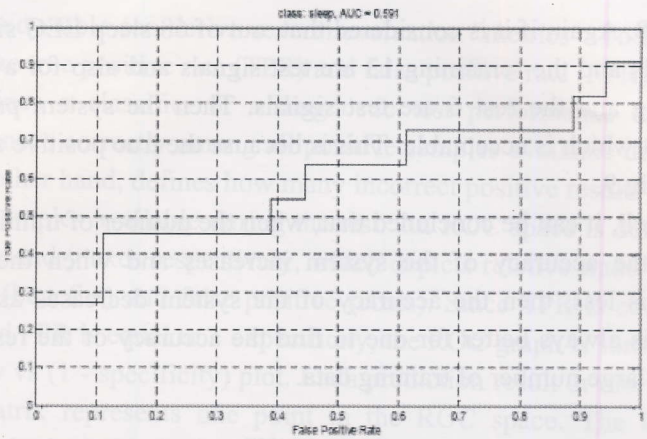

Figure 8: ROC curve considering 50 signals as test signals.

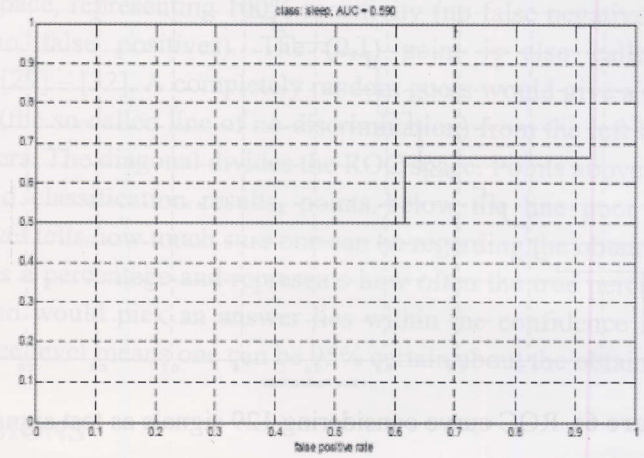

Figure 9: ROC curve considering 55 signals as test signals.

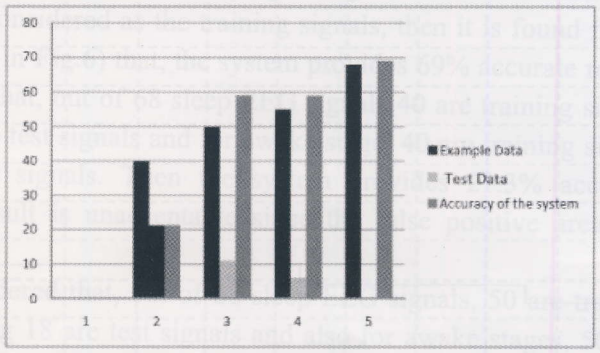

Figure 10: Variation of accuracy of the system with the change in the number of training signals (example data).

6 CLINICAL CORRELATION

The sleep EEG signals can be used to identify disorders and abnormalities. The healthy sleep EEG patterns are matched with the patterns under scrutiny and can be analyzed for abnormalities. Various kinds of neurological diseases can be identified with the help of EEG signals. Sleep disorders, sleep apnea, mental distress, epilepsy, tumors, cerebrovascular and other brain lesions etc. are a few prominent names. Significant research activities are going around the world to develop these techniques. With the improvement of the biomedical signal acquisition tools and signal processing techniques, it is likely that the effectiveness and accuracy of this kind of analysis will grow up by manifolds.

7 CONCLUSIONS

Biomedical signal processing is one of the flourishing areas of modern science and engineering with EEG signal processing is one of the very important facets of this area. Tremendous research and development activities are going around the world. Medical science in collaboration with modern engineering techniques can provide a massive amount of useful information and solutions in this field. Signal processing techniques are part and parcel of EEG analysis. This paper used the technique of Discrete Wavelet Transform and Fast Fourier Transform to characterize the various stages associated with human sleep. This technique can be useful in extracting features of the sleep EEG signals at a very low cost with the aid of computers. The accuracy of this technique is likely to be raised with the improvement of the biomedical signal acquisition tools, with the development of digital filters and of course with the development of more accurate signal processing algorithms.

REFERENCES

[1] E. Niedermeyer, Lopes da Silva, F., Electroencephalography, 4th Ed., 1999, Williams & Wilkins, Baltimore, MD, 1258 pp.

[2] Atlas of EEG & Seizure Semiology. B. Abou-Khalil; Musilus, K.E.; Elsevier, 2006.

[3] AH Khandoker, CK Karmakar, M Palaniswami, "Interaction between Sleep EEG and ECG Signals during and after Obstructive Sleep Apnea Events with or without Arousals", IEEE Computers in Cardiology 2008; 35:685–688.

[4] Jurysta F, van de Borne P, Migeotte PF, Dumont M, Lanquart JP, Degaute JP, and Linkowski P, "A study of the dynamic interactions between sleep EEG and heart rate variability in healthy young men", Clin Neurophysiol 2003; 114: 2146–2155.

[5] http://www.physionet.org/physiobank/database/sleep-edf

[6] W. B. Meldenson "Human Sleep, Research and Clinical Care" Plenum Medical Book Company New York and London", 1987, pp 6-12.

[7] J. G. Webster "Medical Instrumentation, Application and Design" Third Edition , Wiley 1998, pp 165-171

[8] Robert Grover Brown & Patrick Y.C. Hwang (1997), "Introduction to Random Signals and Applied Kalman Filtering", John Wiley & Sons, ISBN: 0471128392.

[9] "An Introduction to the Theory of Random Signals and Noise", Wilbur B. Davenport and Willian L. Root, IEEE Press, New York, 1987, ISBN: 0-87942-235-1.

[10] Gnanadesikan, R.; Wilk, M.B. (1968). "Probability plotting methods for the analysis of data". Biometrika (Biometrika Trust) 55 (1): 1–17. PMID 5661047.

[11] Macmillan Dictionary for Students Macmillan, Pan Ltd. (1981), page 936. Retrieved 2009-10-1.

[12] E. Niedermeyer, Lopes da Silva, F., Electroencephalography, 4th Ed., 1999, Williams & Wilkins, Baltimore, MD, 1258 .

[13] Guler, I., Kiymik, M. K., Akin, M., & Alkan, A, "AR spectral analysis of EEG signals by using maximum likelihood estimation", Computers in Biology and Medicine, 31, 441–450, 2001.

[14] MacQueen, J. B. (1967). "Some Methods for classification and Analysis of Multivariate Observations". 1. Proceedings of 5th Berkeley Symposium on Mathematical Statistics and Probability. University of California Press. pp. 281–297. MR0214227. Zbl 0214.46201. Retrieved 2009-04-07.

[15] Steinhaus, H. (1956). "Sur la division des corps matériels en parties" (in French). Bull. Acad. Polon. Sci. 4 (12): 801–804. MR0090073. Zbl 0079.16403.

[16] Lloyd, S. P. (1957). "Least square quantization in PCM". Bell Telephone Laboratories Paper. Published in journal much later: Lloyd., S. P. (1982). "Least squares quantization in PCM". IEEE Transactions on Information Theory 28 (2): 129–137. doi:10.1109/TIT.1982.1056489. Retrieved 2009-04- 15.

[17] Aloise, D.; Deshpande, A.; Hansen, P.; Popat, P. (2009). "NP-hardness of Euclidean sum-of-squares clustering". Machine Learning 75: 245–249. doi:10.1007/s10994-009-5103-0.

[18] Dasgupta, S. and Freund, Y. (July 2009). "Random Projection Trees for Vector Quantization". Information Theory, IEEE Transactions on 55: 3229– 3242. doi:10.1109/TIT.2009.2021326.

[19] Inaba, M.; Katoh, N.; Imai, H. (1994). "Applications of weighted Voronoi diagrams and randomization to variance-based k-clustering". Proceedings of 10th ACM Symposium on Computational Geometry. pp. 332–339. doi:10.1145/177424.178042.

[20] Vattani, A. (2009). "k-means requires exponentially many iterations even in the plane". Proceedings of the 25th Symposium on Computational Geometry (SoCG).

[21] Signal detection theory and ROC analysis in psychology and diagnostics : collected papers; Swets, 1996.

[22] J. Fogarty, R. Baker, S. Hudson (2005). "Case studies in the use of ROC curve analysis for sensor-based estimates in human computer interaction". ACM International Conference Proceeding Series, Proceedings of Graphics Interface 2005. Waterloo, Ontario, Canada: Canadian Human-Computer Communications Society.

[23] Fawcett, T. (2006). An introduction to ROC analysis. Pattern Recognition Letters, 27, 861–874.

[24] Hand, D.J., & Till, R.J. (2001). A simple generalization of the area under the ROC curve to multiple class classification problems. Machine Learning, 45, 171-186.

[25] Hanley, JA; BJ McNeil (1983-09-01). "A method of comparing the areas under receiver operating characteristic curves derived from the same cases". Radiology 148 (3): 839–843. PMID 6878708. Retrieved 2008-12-03.

[26] McClish, Donna Katzman (1989-08-01). "Analyzing a Portion of the ROC Curve". Med Decis Making 9 (3): 190–195. doi:10.1177/0272989X8900900307. PMID 2668680. Retrieved 2008-09-29.

[27] Dodd, Lori E.; Margaret S. Pepe (2003). "Partial AUC Estimation and Regression". Biometrics 59 (3): 614–623. doi:10.1111/1541-0420.00071. PMID 14601762. Retrieved 2007-12-18.

[28] D.M. Green and J.M. Swets (1966). Signal detection theory and psychophysics. New York: John Wiley and Sons Inc.. ISBN 0-471-32420-5.

[29] M.H. Zweig and G. Campbell (1993). "Receiver-operating characteristic (ROC) plots: a fundamental evaluation tool in clinical medicine". Clinical chemistry 39 (8): 561–577. PMID 8472349.

[30] M.S. Pepe (2003). The statistical evaluation of medical tests for classification and prediction. New York: Oxford.

[31] N.A. Obuchowski (2003). "Receiver operating characteristic curves and their use in radiology". Radiology 229 (1): 3–8. doi:10.1148/radiol.2291010898. PMID 14519861.

[32] Spackman, K. A. (1989). "Signal detection theory: Valuable tools for evaluating inductive learning". Proceedings of the Sixth International Workshop on Machine Learning. San Mateo, CA: Morgan Kaufmann. pp.160–163.