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Finite Element Simulation of Mixed Convection in a Lid-driven Cavity having Wavy Bottom Surface with Internal Heat Generation

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ABSTRACT

A numerical study has been performed to analyze the mixed convective flow and heat transfer characteristics of heat generating fluids contained in a lid-driven cavity with sinusoidal wavy bottom surface. The vertical walls of the cavity are perfectly insulated while the wavy bottom surface is maintained at a uniform temperature higher than the top lid. The flow is assumed to be two-dimensional. Calculations are carried out through solving governing equations for different parameters by using Galarkin's weighted residual finite element method. The flow pattern and the heat transfer characteristics inside the cavity are presented in the form of streamlines and isotherms for various numbers of undulations λ and heat generating parameter Q. The heat transfer rate is found maximum for the lowest Q at $\lambda = 1$.

Keywords: *Mixed Convection, Wavy Surface, Cavity, Finite Element Method, Heat Generation.*

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1 INTRODUCTION

Mixed convection flow and heat transfer in lid-driven cavities have been receiving a considerable attention in the literature. This attention stems from its importance in vast technological, engineering, and natural applications. Such applications include cooling of electronic devices, lubrication technologies, drying technologies, food processing, float glass production, flow and heat transfer in solar ponds, thermal–hydraulics of nuclear reactors and dynamics of lakes.

Das and Mahmud [1] conducted a numerical investigation of natural convection in an enclosure consisting of two isothermal horizontal wavy walls and two adiabatic vertical straight walls. Adjlout et al. [2] studied laminar natural convection in an inclined cavity with a heated undulated wall, i.e., smooth wave-like pattern. Their results concluded that the hot wall undulation affected the flow and heat transfer rate in the enclosure in which the local Nusselt number distribution resulted in a decrease of heat transfer rate as compared with the square enclosure. Moreover, Kumar [3] conducted a study of flow and thermal field inside a vertical wavy enclosure filled with a porous media. The author illustrated that the surface temperature was very sensitive to the drifts in the surface undulations, phase of the wavy surface and the number of considered waves. It should be pointed out that viscous flow in wavy channels was first analyzed analytically by Bums and Parkes [4], while Goldstein and Sparrow [5] used the naphthalene technique to measure local and average heat transfer coefficients in a corrugated wall channel. Furthermore, Wang and Chen [6] analyzed forced convection in a wavy-wall channel and demonstrated the effects of wavy geometry, Reynolds number and Prandtl number on the skin friction and Nusselt number. Their results illustrated that the amplitudes of skin friction coefficient and Nusselt number increased with an increase in the amplitude to wavelength ratio. Al-Amiri et al. [7] investigated mixed convection heat transfer in a lid-driven cavity with a sinusoidal wavy bottom surface. Their findings were that the corrugated lid-driven cavity could be considered as an effective heat transfer mechanism at larger wavy surface amplitudes and low Richardson numbers. Gustavo et al. [8] analyzed laminar flow through a two-dimensional square cavity with internal (volumetric) heat generation numerically. The problem of unsteady laminar combined forced and free convection flow and heat transfer of an electrically conducting and heat generating or absorbing fluid in a vertical lid-driven cavity in the presence of a magnetic field was formulated by Chamkha [9]. Goutam Saha [10] formulated finite element simulation of magnetoconvection inside a sinusoidal corrugated enclosure with discrete isoflux heating from below. The author discussed the effect of discrete heat source sizes on heat transfer for different values of Grashof number and Hartmann number. Oztop and Bilgen [11] conducted a numerical investigation of natural convection in a differentially heated partitioned square cavity with uniform internal heat generation. Also, Liaqat and Baytas [12] reported a detailed analysis of high-Rayleigh number natural convection flow in a square cavity for Rayleigh numbers from 10^7 to 10^{12}. A numerical study of the effect of conduction through enclosing walls of a square cavity containing volumetric sources was reported by Liaqat and Baytas [13]. Very Recently, Nasrin and Parvin [14] analyzed mixed convection flow and heat transfer in a lid-driven cavity with a sinusoidal wavy bottom surface in presence of transverse magnetic field. Their results illustrated that the average Nusselt number (Nu) at the heated surface increased with an increase of the number of waves as well as the Reynolds number, while decreased with increasing Hartmann number.

To the best of the authors' knowledge, little attention has been paid to the problem of mixed convection flow and heat transfer in a lid-driven cavity that is heated from a wavy bottom surface. The objective of the present study is to examine the momentum and energy transport processes in a lid-driven cavity having wavy bottom surface with internal heat generation. The results are shown in terms of parametric presentations of streamlines and isotherms for various pertinent dimensionless parameters such as heat generating parameter Q and number of undulations λ offered by the wavy bottom surface. Finally, the implications of the above parameters are also depicted on the average Nusselt number.

2 PROBLEM FORMULATION

The treated problem is a two-dimensional square cavity with a side length L. The physical system considered in the present study is displayed in **Fig.1**. The vertical walls are considered adiabatic and impermeable while the horizontal walls are maintained at uniform but different temperatures such that the bottom wall is assigned to temperature T_{hot} while the top wall is subjected to temperature T_{cold}. Under all circumstances $T_{hot} > T_{cold}$ condition is maintained. Furthermore, the top wall is assumed to slide from left to right at a constant speed U_i.

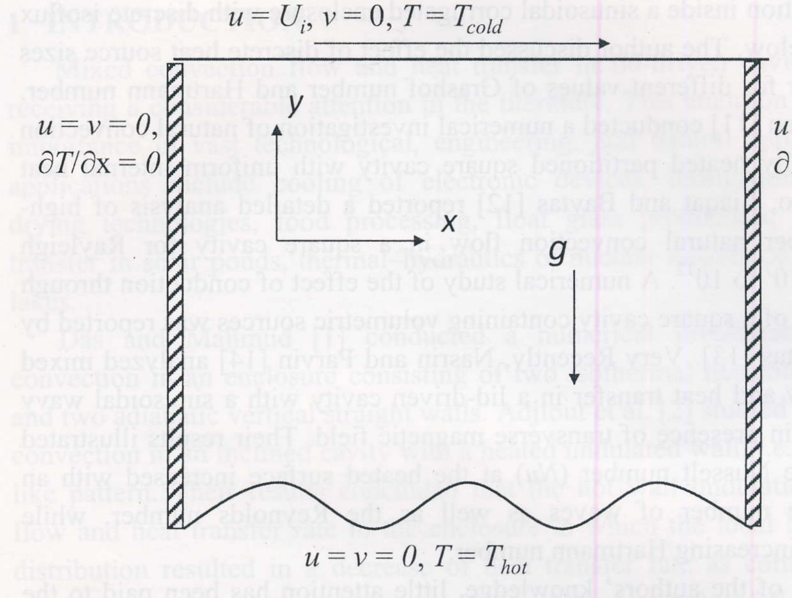

Figure 1: Schematic diagram of the problem.

3 MATHEMATICAL FORMULATION

The working heat generating fluid is assumed to be Newtonian and incompressible with the flow is set to operate in the laminar mixed convection regime. The governing equations under Boussinesq approximation are as follows:

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (1)$$

Momentum Equations

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad \ldots \qquad (2)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\beta\left(T - T_c\right) \qquad (3)$$

Energy Equations

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho\, c_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{q}{\rho\, c_p} \qquad \ldots \quad \ldots \qquad (4)$$

The boundary conditions for the present problem can be written as follows:

at the sliding lid: $u = U_i, v = 0, T = T_{cold}$,

at all vertical walls: $u = 0, v = 0, \dfrac{\partial T}{\partial n} = 0$,

at the wavy bottom surface: $u = 0,\ v = 0,\ T = T_{hot}$

The rate of heat transfer is computed at the bottom wall and is expressed in terms of the local Nusselt number as $Nu_{local} = -\dfrac{\partial T}{\partial n} L$ where, h represents the heat transfer coefficient, k thermal conductivity and n the coordinate direction normal to the surface. The dimensionless normal temperature gradient can be written as

$$\frac{\partial T}{\partial n} = \sqrt{\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2}$$

while the average Nusselt number (Nu) is obtained by integrating the local Nusselt number along the bottom wavy surface and is defined by

$$Nu = \frac{1}{L_S} \int_0^{L_S} Nu_{local}\, dn$$

where L_S is the total chord length of the wavy surface and n is the coordinate along the wavy surface.

To make the equations (1) – (4) dimensionless, the following dimensionless variables are used:

$$X = \frac{x}{L},\quad Y = \frac{y}{L},\quad U = \frac{u}{U_i},\quad V = \frac{v}{U_i},\quad P = \frac{p}{\rho U_i^2},\quad \theta = \frac{T - T_{cold}}{T_{hot} - T_{cold}}$$

where X and Y are the coordinates varying along horizontal and vertical directions respectively, U and V are the velocity components in the X and Y directions respectively, θ is the dimensionless temperature and P is the dimensionless pressure.

Upon incorporating dimensionless parameters the equations (1) – (4) become:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{Ra}{Re^2\,Pr}\theta$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{RePr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right)+Q$$

where $Re=\frac{U_i L}{\nu}$, $Pr=\frac{\nu}{\alpha}$ and $Ra=\frac{g\beta\left(T_{hot}-T_{cold}\right)L^3}{\nu\alpha}$ are Reynolds number, Prandtl number and Rayleigh number respectively and $Q=\frac{qL^2}{\nu\left(T_{hot}-T_{cold}\right)}$ is the heat generating parameter. The shape of the bottom wavy surface profile is assumed to mimic the following pattern $Y=A\left[1-\cos\left(2\lambda\pi X\right)\right]$ where A is the dimensionless amplitude of the wavy surface and λ is the number of undulations.

The nondimensional boundary conditions for the present problem are specified as follows:

at the sliding lid: $U=1, V=0, \theta=0$,

at all solid vertical side walls: $U=V=0, \frac{\partial\theta}{\partial X}=0$,

at the wavy bottom surface: $U=V=0, \theta=1$

The dimensionless normal temperature gradient can be written as

$$\frac{\partial\theta}{\partial N}=\frac{1}{L}\sqrt{\left(\frac{\partial\theta}{\partial X}\right)^2+\left(\frac{\partial\theta}{\partial Y}\right)^2}$$

while the average Nusselt number (Nu) takes the form

$$Nu=\frac{1}{S}\int_0^S\frac{\partial\theta}{\partial N}dN$$

where S and N are the non dimensional length and coordinate along the wavy surface.

4 COMPUTATIONAL PROCEDURE

The numerical procedure used in this work is based on the Galerkin weighted residual method of finite element formulation. The application of this technique was well described by Taylor and Hood [15] and Dechaumphai [16]. At first, the solution domain is discretized into finite element meshes, which are composed of non-uniform triangular elements. Then the nonlinear governing partial differential equations (i.e. mass, momentum and energy equations) are transferred into a system of integral equations by applying Galerkin weighted residual method. The integration involved in each term of these equations is performed by using Gauss's quadrature method. The nonlinear algebraic equations so obtained are modified by imposition of boundary conditions. Applying Newton's method these modified nonlinear equations are transferred into linear algebraic equations. Finally, these linear equations are solved by using Triangular Factorization method.

4.1 Validation of code

The model validation is an essential part of a numerical investigation. Hence, the outcome of the present numerical code is benchmarked against the numerical results of Al-Amiri et al. [7] which were reported for laminar mixed convection heat transfer in a lid-driven cavity heated from below. The comparison is conducted while employing the following dimensionless parameters: Re = 500, Ri = 0.4 and Pr = 1. Excellent agreement is achieved, as illustrated in **Fig.2**,

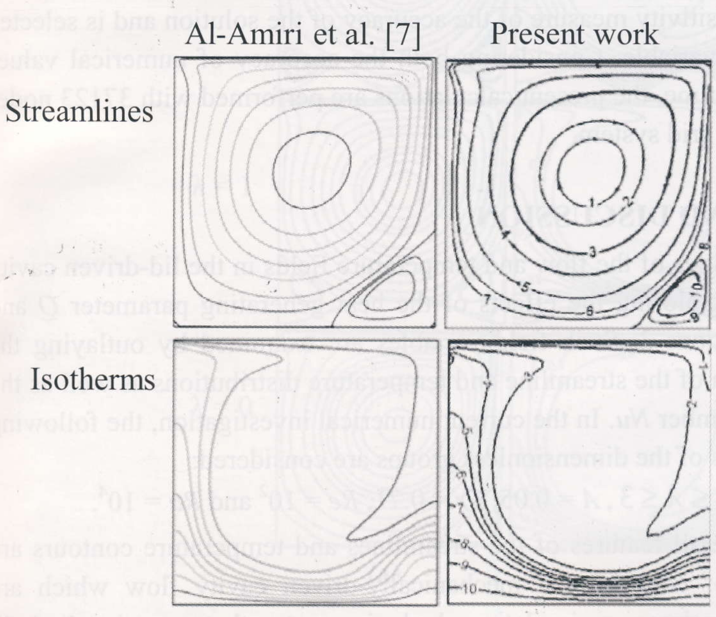

Figure 2: Comparison of the streamlines and isotherms between the present work and Al-Amiri et al. [7] at $Re = 500$, $Ri = 0.4$ and $Pr = 1$.

Table 1: Grid Sensitivity Check at $Q = 10$, $Pr = 0.71$, $Ra = 10^4$, $Re = 100$, $A = 0.05$ and $\lambda = 3$

Nodes (elements)	4931 (716)	10853 (1612)	18916 (2832)	37123 (5604)	47853 (7244)
Nu	1.578491	1.796590	1.798085	2.006512	2.006512
Time (s)	236.265	298.594	390.157	430.328	647.375

between present results and the numerical results of Amiri [7] for both the streamlines and isotherms inside the cavity. These validations boost the confidence in our numerical code to carry on with the above stated objectives of the current investigation.

4.2 Grid Sensitivity Test

In order to determine the proper grid size for this study, a grid independence test is conducted with five types of mesh for Q = 10, Pr = 0.71, Ra = 10^4, A = 0.05, Re = 100 and λ = 3 which is shown in **Table.1**. The extreme value of Nu is used as a sensitivity measure of the accuracy of the solution and is selected as the monitoring variable. Considering both the accuracy of numerical values and computational time, the present calculations are performed with 37123 nodes and 5604 elements grid system.

5 RESULTS AND DISCUSSION

The characteristics of the flow and temperature fields in the lid-driven cavity are examined by exploring the effects of the heat generating parameter Q and number of undulations λ. Such field variables are examined by outlaying the steady state version of the streamline and temperature distributions as well as the average Nusselt number Nu. In the current numerical investigation, the following parametric domains of the dimensionless groups are considered:

$$0 \le Q \le 20,\ 0 \le \lambda \le 3,\ A = 0.05,\ Pr = 0.71,\ Re = 10^2 \text{ and } Ra = 10^4.$$

For Q = 0, the overall features of the streamlines and temperature contours are similar to those of conventional mechanically-driven cavity flow which are characterized by a primary recirculating clockwise vortex that occupies the bulk of the cavity. In addition, minor vortices tend to form near the bottom corners, which are increased in numbers with elevating λ. Meanwhile, thinner boundary layers are depicted to form near the bottom wall. This is attributed to the increase in the contribution of convection heat transfer mechanism, which causes steep temperature gradients in the vertical direction near the bottom wall as shown in **Fig.3** (a), **Fig.3** (b). It appears from the figures that varying the number of undulations λ between 0 and 3 does not disturb the global flow and isotherm patterns except in the vicinity of the bottom wall, where the contour lines mimic the wall's profile.

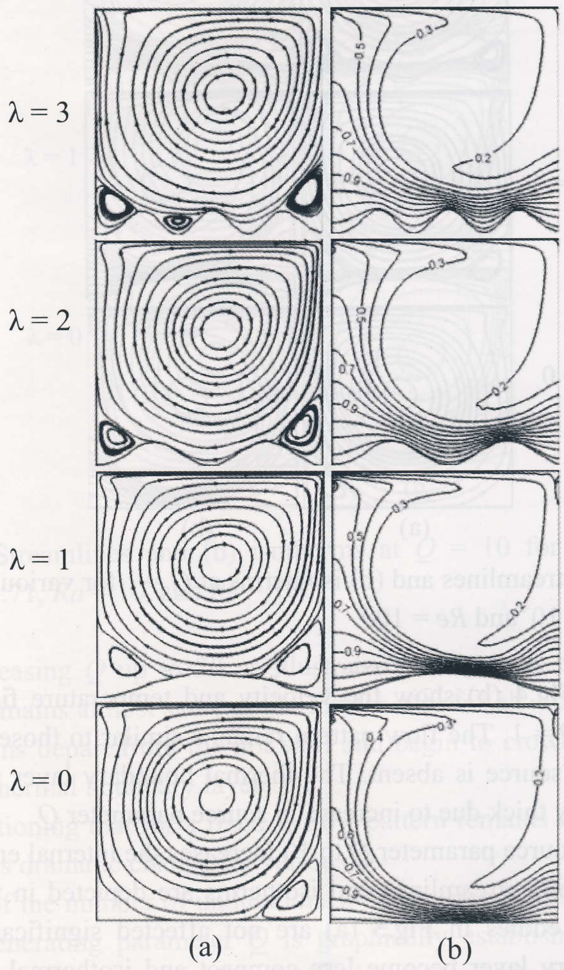

Figure 3: (a) Streamlines and (b) isotherms at $Q = 0$ for various λ with $A = 0.05$, $Pr = 0.71$, $Ra = 10^4$ and $Re = 100$.

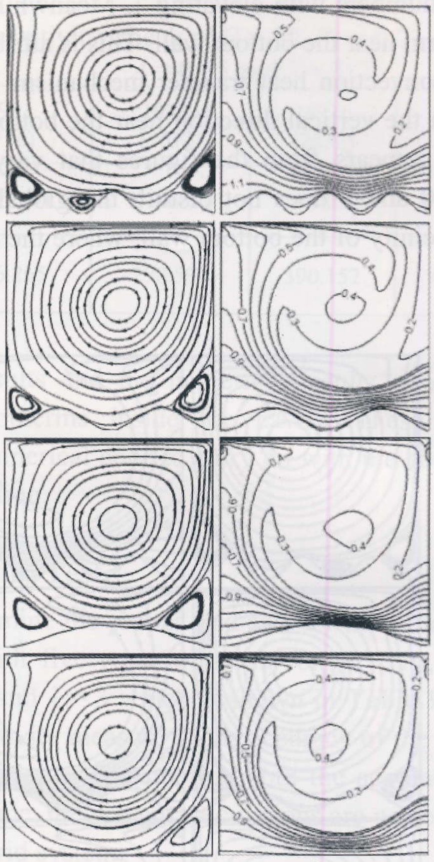

Figure 4: (a) Streamlines and (b) isotherms at $Q = 1$ for various λ with $A = 0.05$, $Pr = 0.71$, $Ra = 10^4$ and $Re = 100$.

Fig.4 (a) and **Fig.4** (b) show the velocity and temperature fields for different values of λ at $Q = 1$. The flow pattern remains similar to those of previous case where the heat source is absent. The thermal boundary layer near the hot wall becomes slightly thick due to increase in source parameter Q.

Increasing the source parameter Q to 10 increases the internal energy of the fluid. The corresponding streamlines and isotherms are depicted in the **Fig.5** (a) and **Fig.5** (b). The eddies in **Fig.5** (a) are not affected significantly whereas the thermal boundary layer become less compact and isothermal lines occupy the whole cavity. Elevation in λ increases the number of vortices near the wavy surface in the streamline and the temperature profile takes the wavy pattern in the neighborhood of the bottom surface.

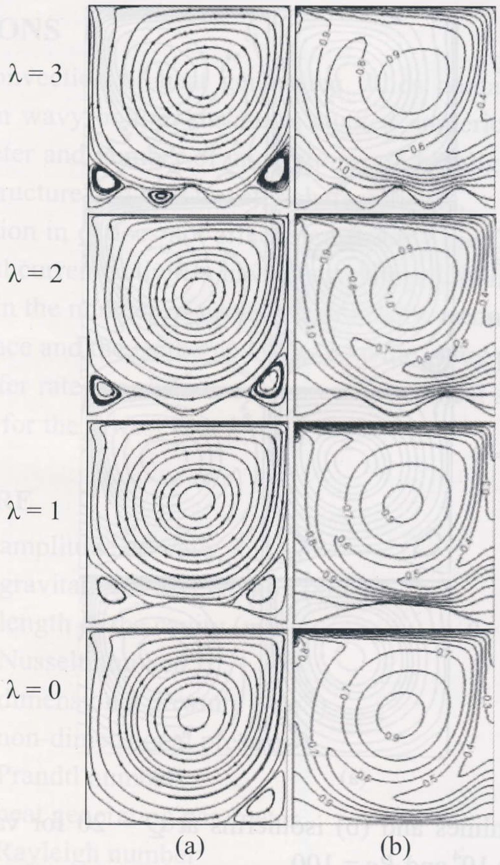

Figure 5: (a) Streamlines and (b) isotherms at Q = 10 for different λ with $A = 0.05$, $Pr = 0.71$, $Ra = 10^4$ and $Re = 100$.

On further increasing Q up to 20 as illustrated in **Fig.6** (a) and **Fig.6** (b) the velocity field remains almost same as the former case for all λ but it is noticeable that the isotherms depart from the hot wall and begin to crowd near the top lid forming a thin thermal boundary layer.

It is worth mentioning that for a fixed λ, flow pattern remains almost similar but thermal field has dramatic change for various Q.

The influence of the number of undulations λ on the average Nusselt number for various heat generating parameter Q is graphically established in **Fig.7**. The result exhibits that the average Nusselt number is found to increase for $\lambda \le 1$, then decrease between $1 < \lambda \le 2$ and becomes almost flatten from $\lambda = 2$ to $\lambda = 3$. Moreover, the average Nusselt number is reduced with escalating Q.

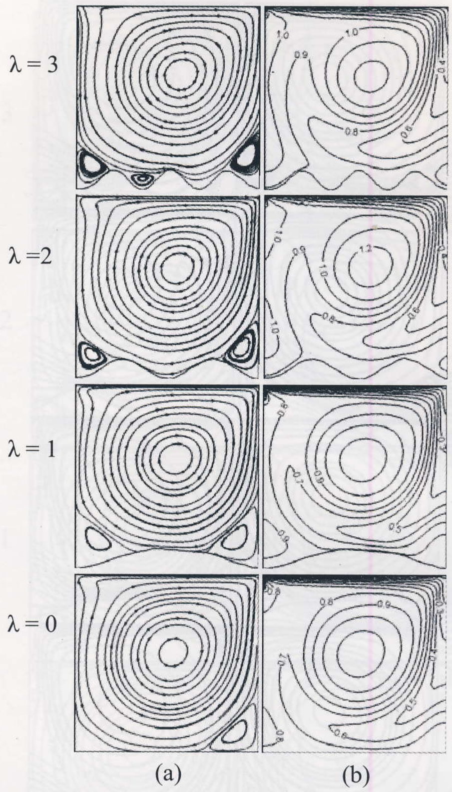

Figure 6: (a) Streamlines and (b) isotherms at Q = 20 for various λ with A = 0.05, Pr = 0.71, Ra = 10^4 and Re = 100.

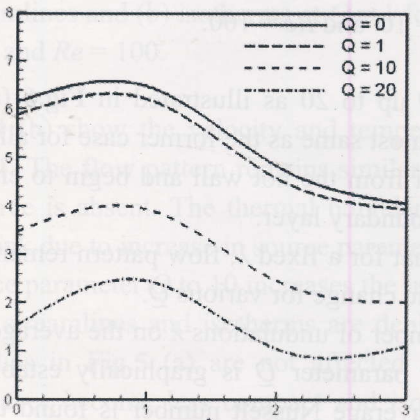

Figure 7: Effect of λ on Nu for different Q while A = 0.05, Pr = 0.71, Ra = 10^4 and Re = 100.

6 CONCLUSIONS

The mixed convection of heat generating fluids contained in a lid-driven cavity heated from wavy bottom surface is studied numerically. Effects of heat generation parameter and number of undulations are highlighted to explore their impacts on flow structure and heat transfer characteristics.

(i) The variation in Q does not affect significantly the flow pattern however the thermal current activities have been changed dramatically.

(ii) The raise in the number of waves creates more secondary eddies near the wavy surface and the isothermal lines take the wavy profile.

(iii) Heat transfer rate is reduced in the presence of heat sources and is found maximum for the lowest Q at $\lambda = 1$.

NOMENCLATURE

A	amplitude of wavy surface
g	gravitational acceleration (ms^{-2})
L	length of the cavity (m)
Nu	Nusselt number
p	dimensional pressure (Nm^{-2})
P	non-dimensional pressure
Pr	Prandtl number
Q	heat generating parameter
Ra	Rayleigh number
Re	Reynolds number
T	dimensional temperature (K)
u, v	velocity components (ms^{-1}) along x, y direction
U, V	dimensionless velocity components along X, Y direction
U_i	lid velocity (ms^{-1})
x, y	Cartesian coordinates (m)
X, Y	non-dimensional Cartesian coordinates
Greek symbols	
α	thermal diffusivity (m^2s^{-1})
β	thermal expansion coefficient (K^{-1})
θ	non-dimensional temperature
λ	number of undulations
μ	dynamic viscosity of the fluid ($Kg\ m^{-1}s^{-1}$)
ν	kinematic viscosity of the fluid (m^2s^{-1})
ρ	density of the fluid ($Kg\ m^{-3}$)

Subscripts

cold	less heated wall
hot	heated wall

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